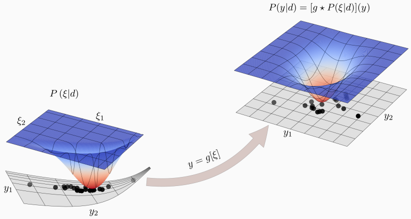

Signal reconstruction for fields

USING PROBABILISTIC REASONING & FORWARD MODELING

Philipp Frank[1]

IA-FORTH seminar: Institute of Astrophysics - FORTH, University of Crete, Greece,
September 13, 2023

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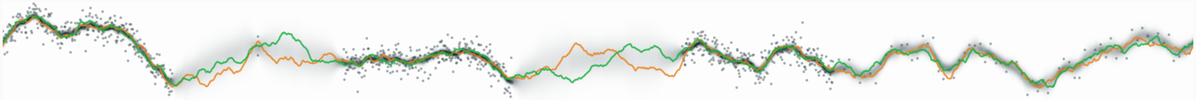
(1) Max-Planck Institute for Astrophysics MPA, Garching, Germany



Signal reconstruction for fields - Overview

- Why?
  - Signal based estimators
  - Probabilistic reasoning
  - Forward modeling

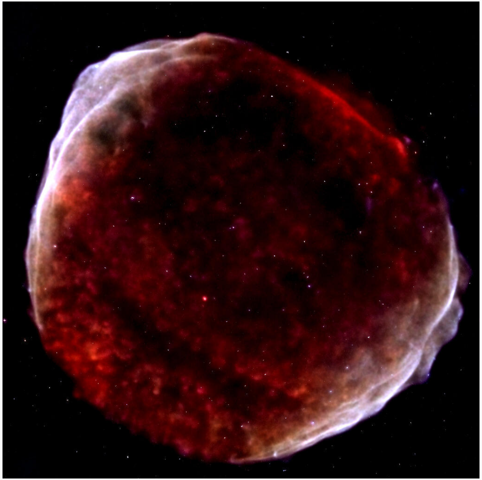


Figure 1: SN1006 from Chandra data [WEG+23]

Signal reconstruction for fields - Overview

- Why?
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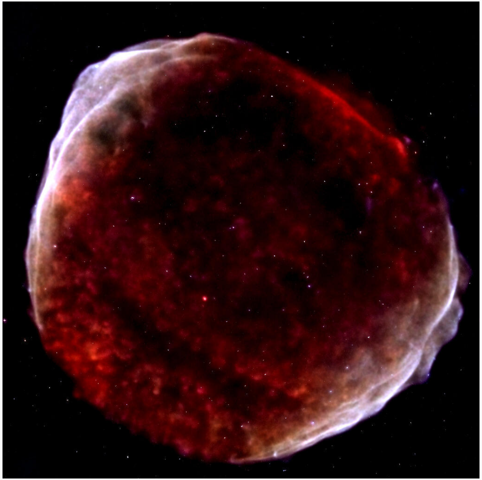
- How?
  - Generative models
  - Gaussian processes
  - Likelihood & Instrument models
  - Approximate inference



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- What?
  - Radio interferometry - VLBI imaging
  - Faraday sky & LOS magnetic field
  - GAIA 3D dust tomography
  - Chandra/Erosita X-ray imaging
  - Fermi γ-ray sky
  - ...

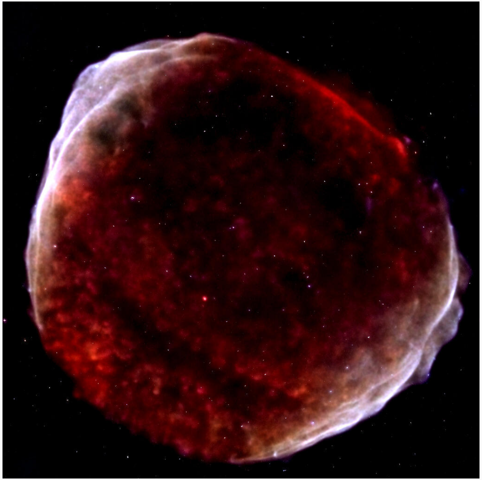


Figure 1: SN1006 from Chandra data [WEG$^+$23]

Why?

Why? - Signal based estimators

Given observational data $d \rightarrow$ Obtain answers a to questions q /

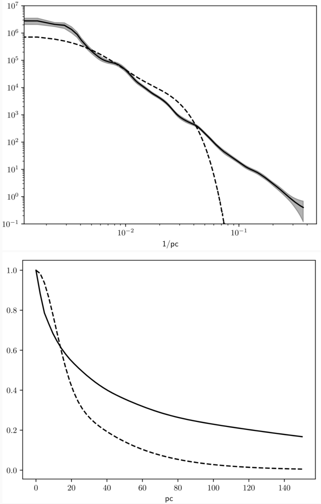
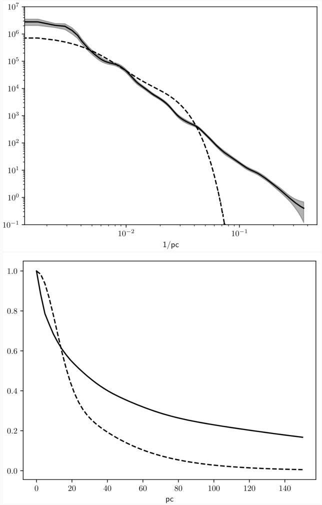


Figure 2: [LGE20]

Why? - Signal based estimators

Given observational data $d \rightarrow$ Obtain answers a to questions q / obtain estimators $\hat{a} = g_q(d, M)$ under model assumptions M.



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For example, given GAIA extinction data ...

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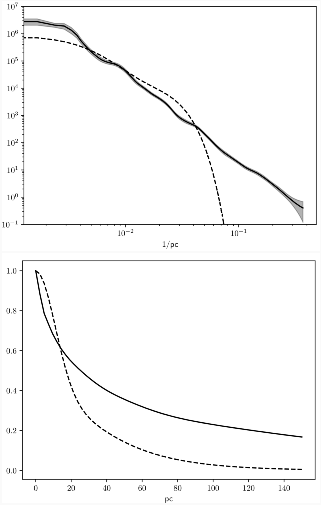

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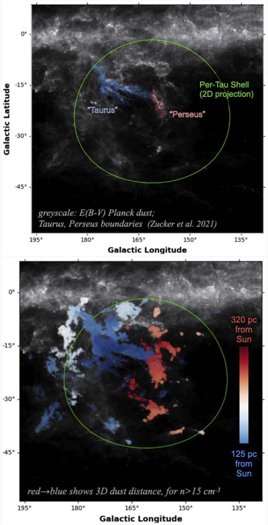

Figure 2: [BZG+21]

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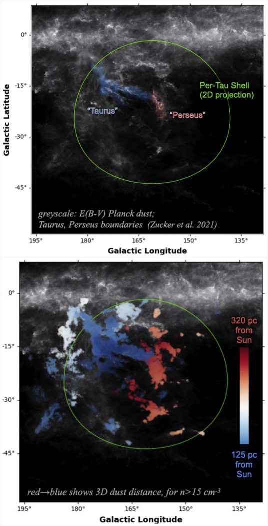
Given signal $s \equiv$ complete description of the system: $(\rho_{\mathrm{dust}}, \mathrm{T}_{\mathrm{dust}}, \ldots)$



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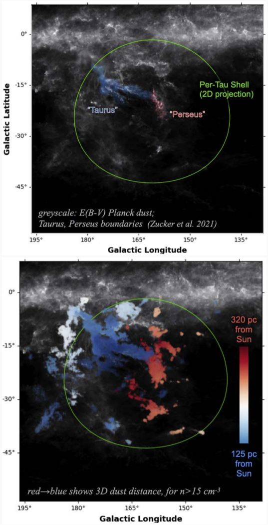
Signal based answers

$$a = q(s) \ .$$

With: $s =$ signal, $q =$ question.



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Why? - Probabilistic reasoning

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$$\hat{a} = \langle q(s) \rangle_{\mathcal{P}(s|d,M)} \equiv \int q(s) \ \mathcal{P}(s|d,M) \ \mathrm{d}s$$

With: $s =$ signal, $q =$ question, $\mathcal{P}(s|d,M) =$ posterior distribution, $M =$ Model.

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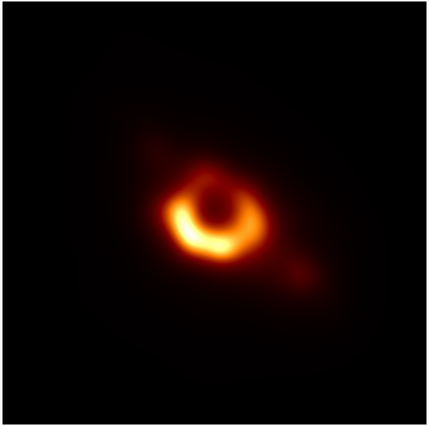
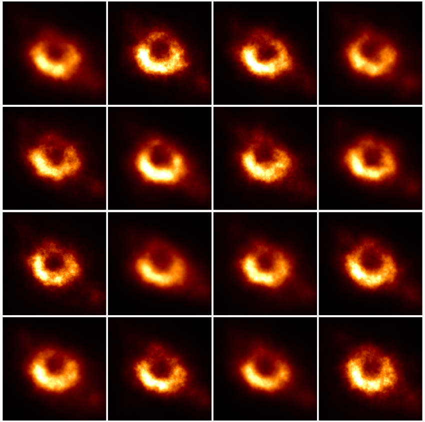


[AFH+22]



Why? - Probabilistic reasoning & Forward modeling

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Product rule aka Bayes' Theorem

$$\mathcal{P}\left(s|d,M\right) = \frac{\mathcal{P}\left(s,d|M\right)}{\int \mathcal{P}\left(s,d|M\right)\ \mathrm{d}s}\ .$$

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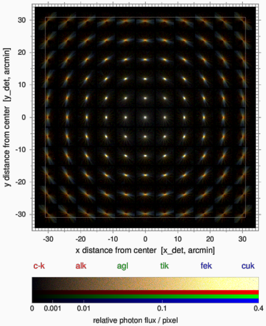
$$\mathcal{P}\left(s, d|M\right) = \underbrace{\mathcal{P}\left(d|s, M\right)}_{\text{Likelihood}}\ \underbrace{\mathcal{P}\left(s|M\right)}_{\text{Prior}}\ .$$

Likelihood $\leftrightarrow$ Instrument model

- Signal $\rightarrow$ Observable
- Instrument geometry
- Exposure
- Point-spread function
- Noise processes
- ...



[PAAe21]

How? - Gaussian & generative processes

- Probability distributions $\mathcal{P}\left(s|M\right)$ over functions $s_x \equiv s(x)$, with $s \in \mathcal{L}^2\left[\Omega\right],\ x \in \Omega \subset \mathbb{R}^N$.

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- Mean field: $m_x = m(x) \equiv \langle s_x \rangle_{\mathcal{P}(s)}$.
- Correlation structure: $C_{xy} = C(x,y) \equiv \left\langle \left(s_x - m_x\right)\left(s_y - m_y\right)^* \right\rangle_{\mathcal{P}(s)}$.

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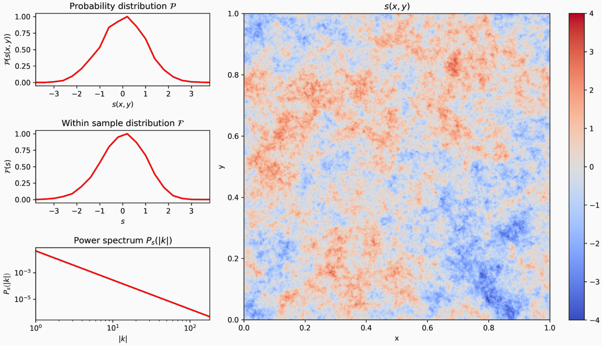
Gaussian process (GP) distribution

$$\mathcal{P}(s|m, C) = \mathcal{N}\left(s; m, C\right) = \frac{1}{\sqrt{|2\pi C|}} \exp\left(-\frac{1}{2}(s - m)^\dagger C^{-1}(s - m)\right)$$
$$= \frac{1}{\sqrt{|2\pi C|}} \exp\left(-\frac{1}{2}\int\int (s(x) - m(x))\ C^{-1}(x, y)\ (s(y) - m(y))\ \mathrm{d}x\mathrm{d}y\right)$$

Inverse operator: C^{-1}; Functional determinant: $|\bullet|$.

How? - Gaussian & generative processes

$$\text{Fields,} \quad \text{with} \quad C_{xy} \propto |\vec{x} - \vec{y}|^{-\alpha} \ .$$



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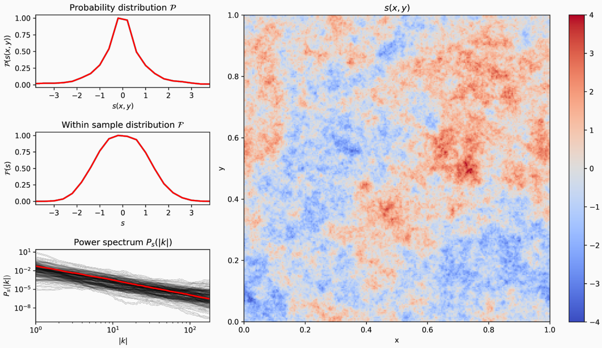

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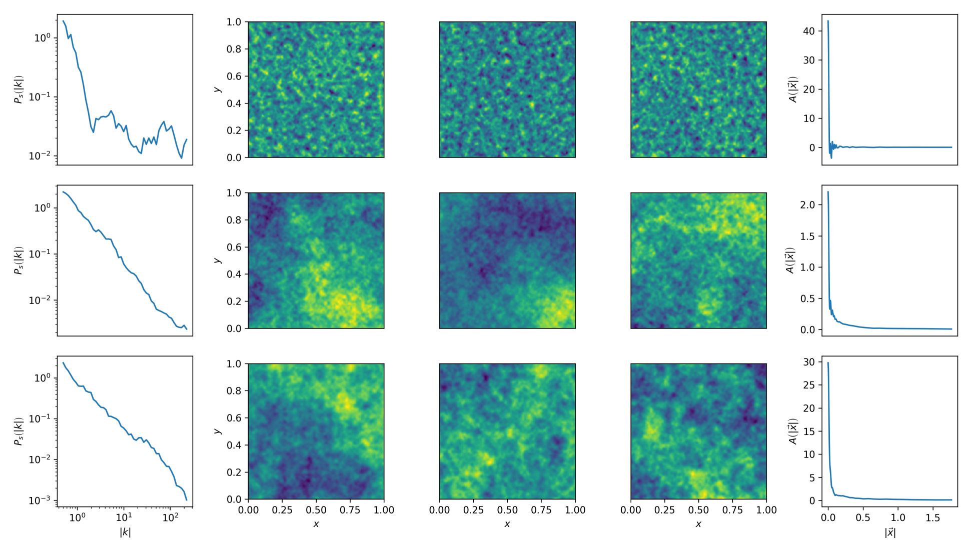
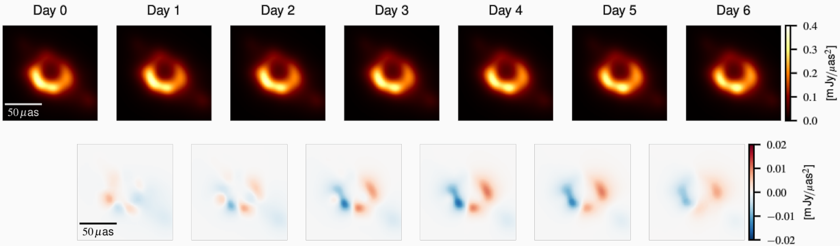
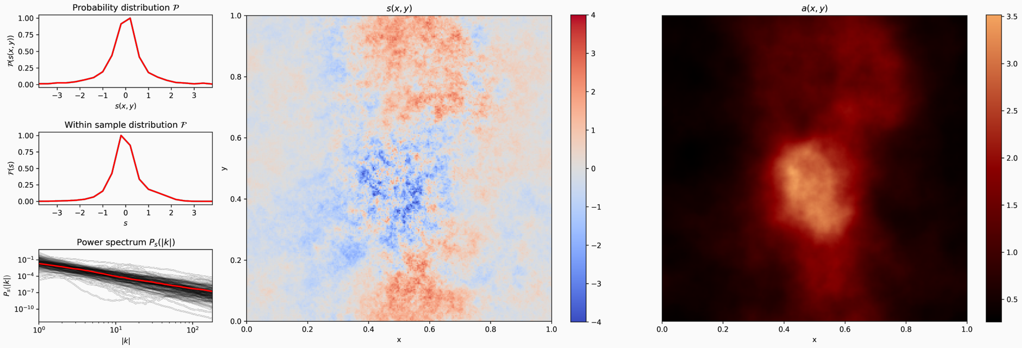


How? - VLBI of M87* [AFH+22]



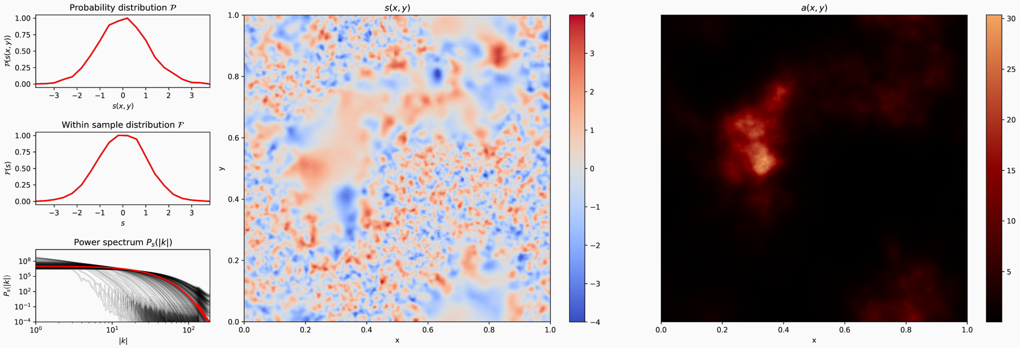
How? - Gaussian & generative processes

$$\text{Fields, with } C_{xy} \propto a(\vec{x})\, C_{P_s}(|\vec{x}-\vec{y}|)\, a(\vec{y}) \;.$$



How? - Gaussian & generative processes

Fields, with $C_{xy} \propto C_{a}(\vec{x}, \vec{y})$.



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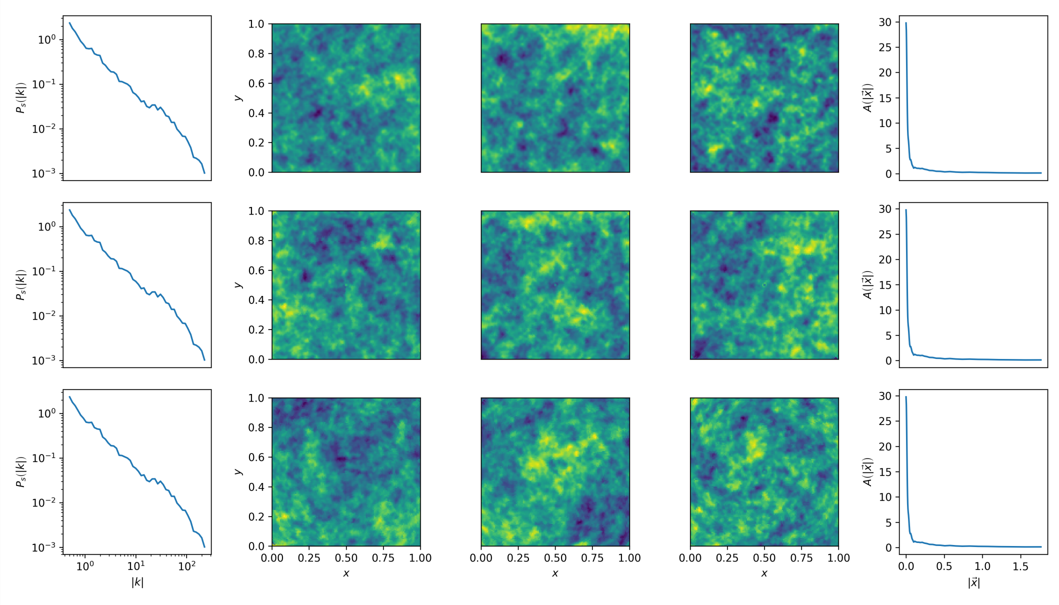

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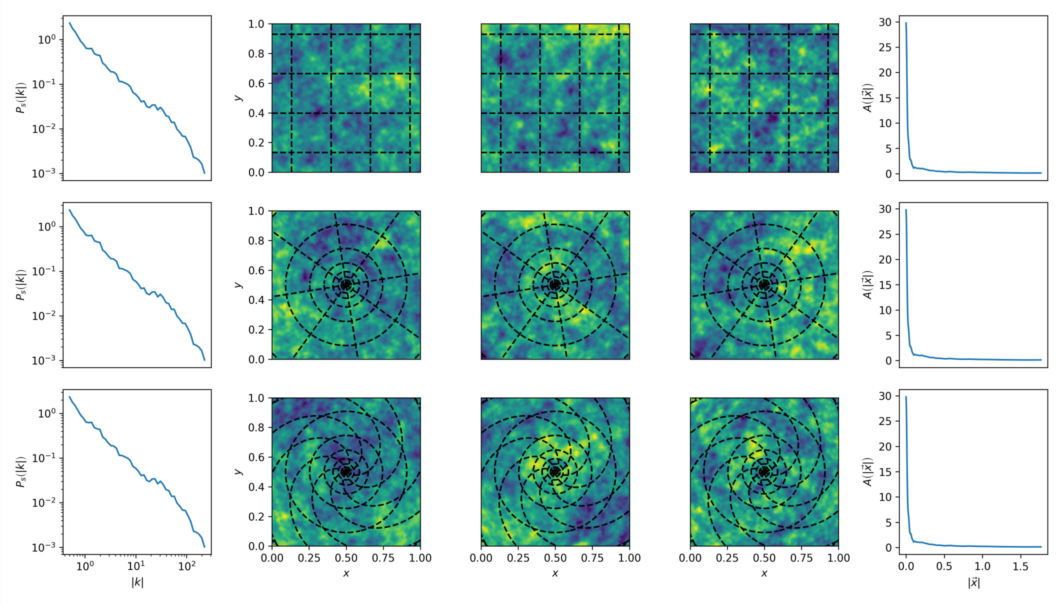

How? - Approximate inference

Signal based estimators

$$\hat{a} = \langle q\left(s\right) \rangle_{\mathcal{P}(s|d,M)} \equiv \int q\left(s\right)\ \mathcal{P}\left(s|d,M\right)\ \mathrm{d}s \qquad .$$

With: $s =$ signal, $q =$ question, $\mathcal{P}\left(s|d,M\right) =$ posterior distribution.

How? - Approximate inference

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$$\hat{a} = \langle q(s) \rangle_{\mathcal{P}(s|d,M)} \equiv \int q(s)\ \mathcal{P}(s|d,M)\ \mathrm{d}s \approx \int q(s)\ \mathcal{Q}(s|d,M)\ \mathrm{d}s$$

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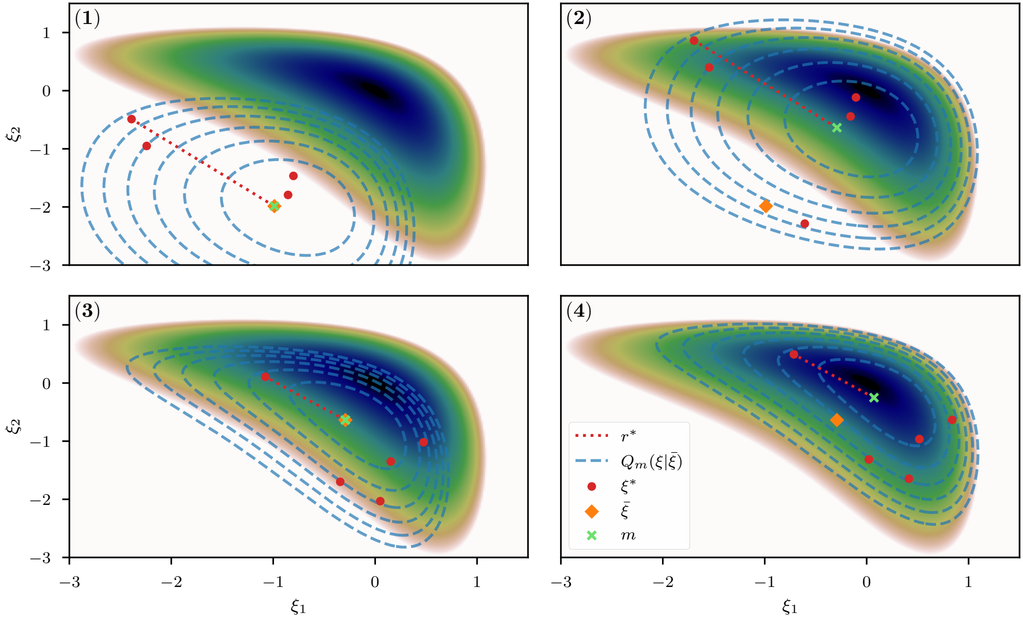
With: $s =$ signal, $q =$ question, $\mathcal{P}(s|d,M) =$ posterior distribution.

Kullback-Leibler divergence

$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = -\int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right)\ \mathcal{Q}_\sigma(\xi)\ \mathrm{d}\xi$$

Posterior: $\mathcal{P}(\xi|d,M)$; Approximation: $\mathcal{Q}(\xi|d,M)$; Variational parameters: σ.

How? - Geometric Variational Inference (geoVI) [FLE21]



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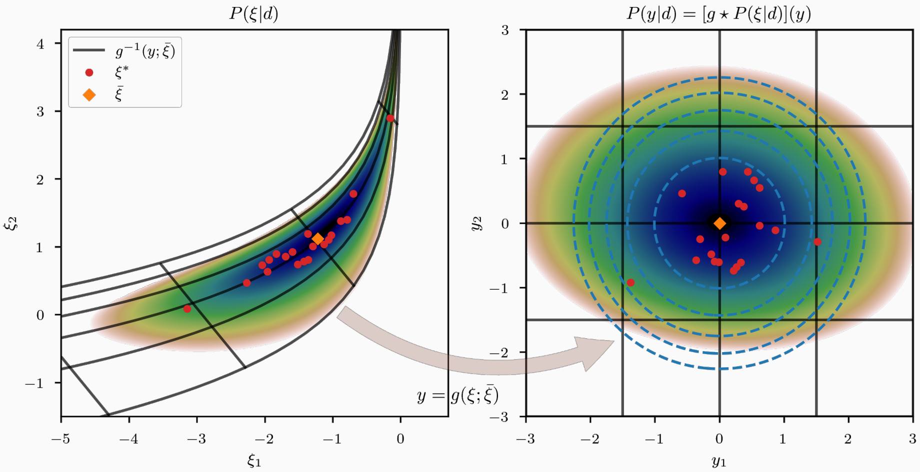
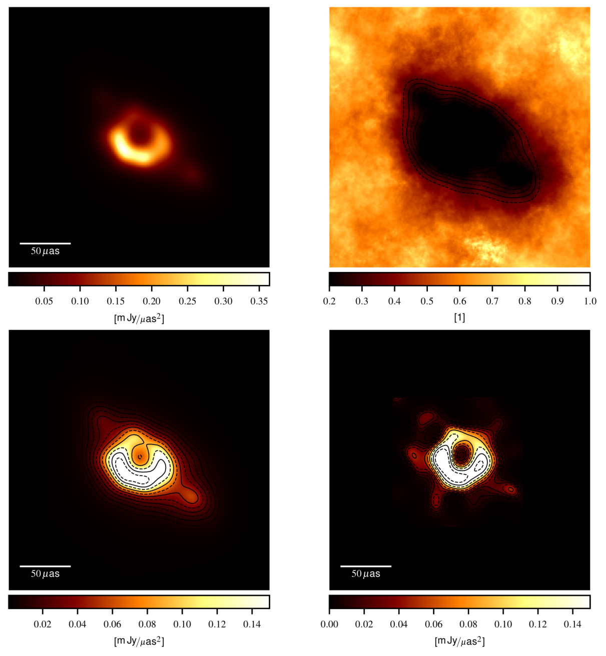


What?

Radio interferometry - VLBI imaging [AFH+22]



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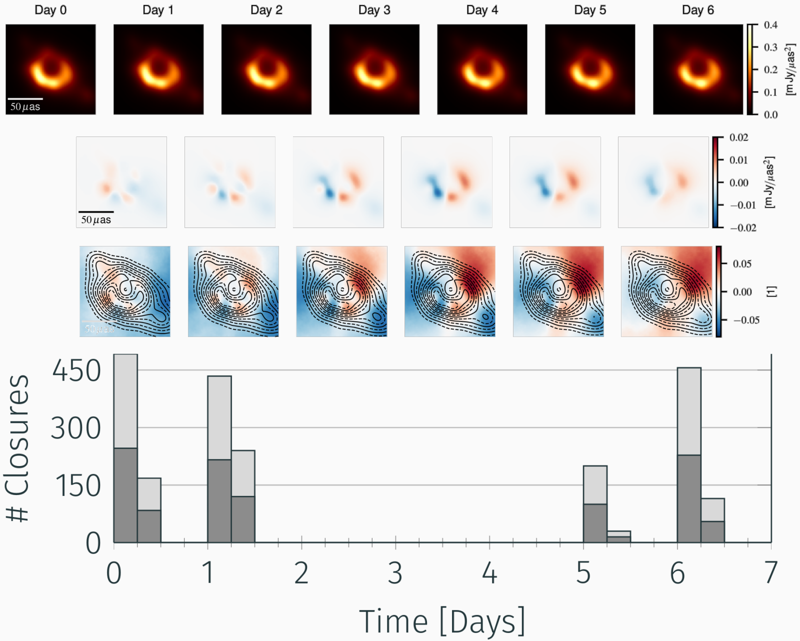
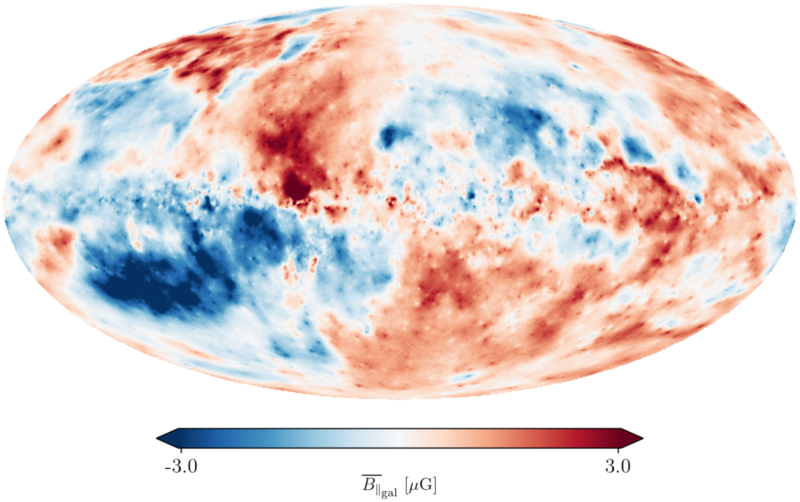


Ring fitting (see [?, Table 7])

	d (μas)	w (μas)	η ($^\circ$)	A	f_C
EHT-IMAGING [?]					
April 5	39.3 ± 1.6	16.2 ± 2.0	148.3 ± 4.8	0.25 ± 0.02	0.08
April 6	39.6 ± 1.8	16.2 ± 1.7	151.1 ± 8.6	0.25 ± 0.02	0.06
April 10	40.7 ± 1.6	15.7 ± 2.0	171.2 ± 6.9	0.23 ± 0.03	0.04
April 11	41.0 ± 1.4	15.5 ± 1.8	168.0 ± 6.9	0.20 ± 0.02	0.04
OUR METHOD					
UNCERTAINTY AS PER [?, TABLE 7])					
April 5	44.4 ± 3.4	23.2 ± 5.2	164.9 ± 9.5	0.26 ± 0.04	0.365
April 6	44.4 ± 2.9	23.3 ± 5.4	161.7 ± 5.6	0.24 ± 0.04	0.374
April 10	44.8 ± 2.8	23.0 ± 5.0	176.7 ± 9.8	0.22 ± 0.03	0.374
April 11	44.6 ± 2.8	22.8 ± 4.8	180.1 ± 10.4	0.22 ± 0.03	0.372
SAMPLE UNCERTAINTY					
April 5	44.1 ± 1.2	23.1 ± 2.4	163.9 ± 5.0	0.25 ± 0.03	0.377 ± 0.081
April 6	44.0 ± 1.2	22.9 ± 2.4	161.9 ± 6.0	0.24 ± 0.03	0.385 ± 0.085
April 10	44.6 ± 1.2	22.9 ± 2.5	176.2 ± 6.5	0.22 ± 0.03	0.383 ± 0.089
April 11	44.6 ± 1.2	23.0 ± 2.6	179.8 ± 6.2	0.22 ± 0.03	0.383 ± 0.090

Faraday sky & LOS magnetic field [HHF+23]



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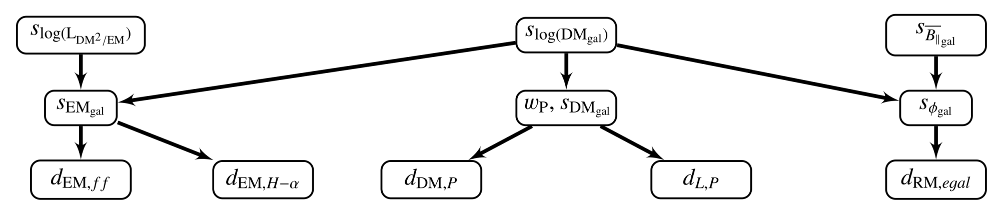

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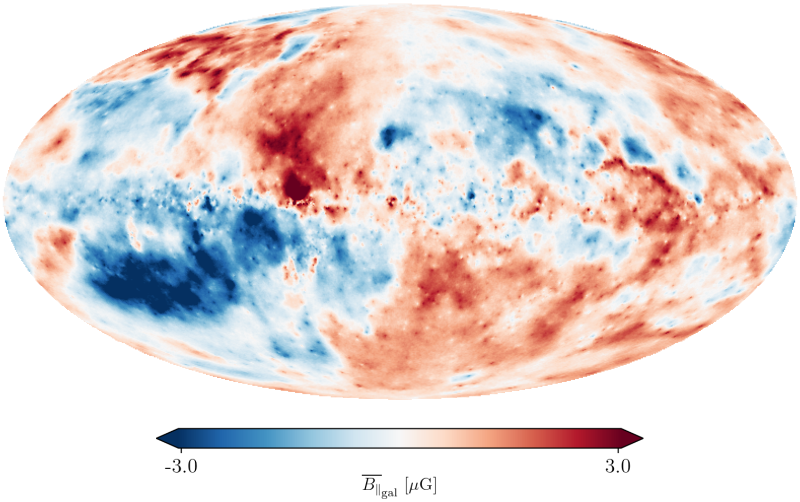

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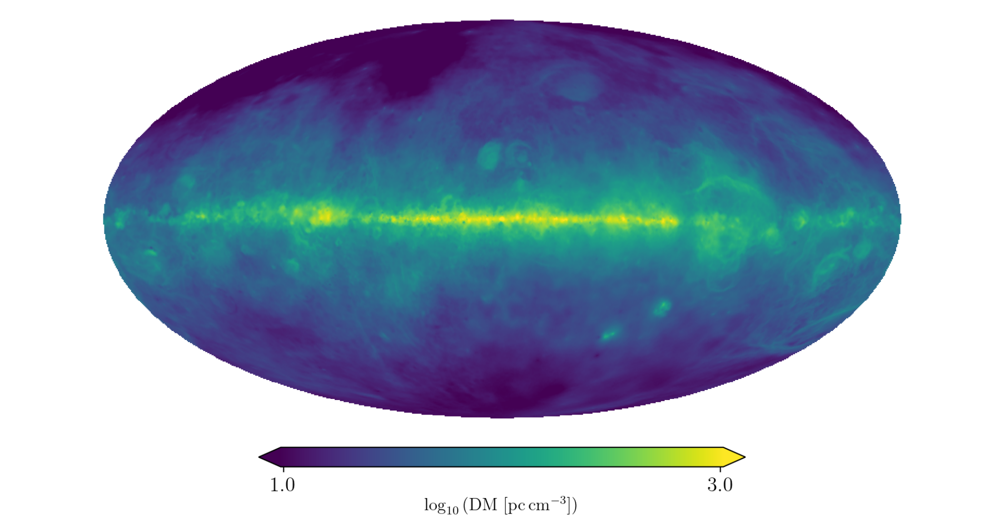

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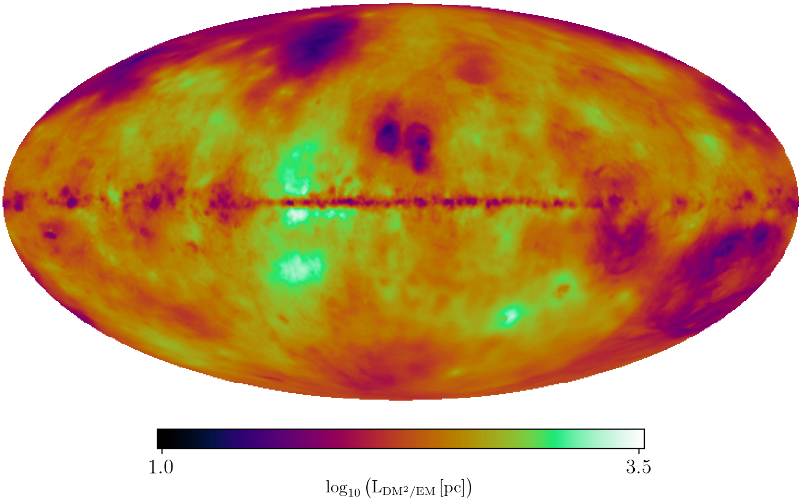
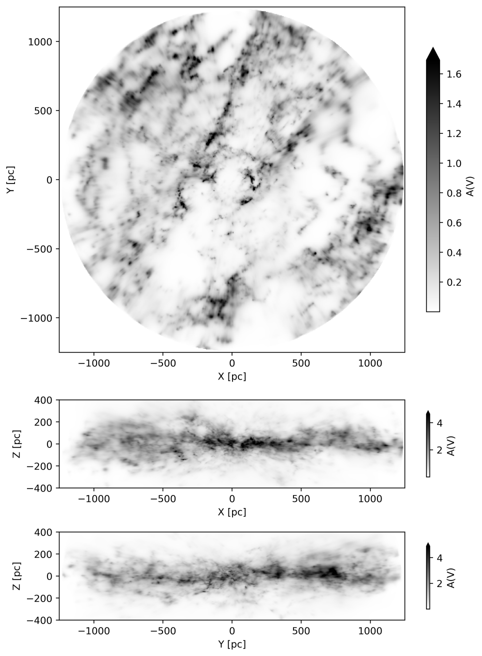


GAIA 3D dust tomography [EZF^{+}23]



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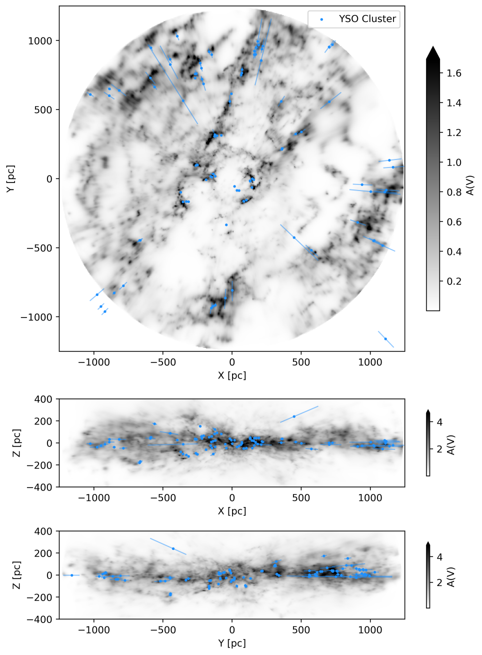

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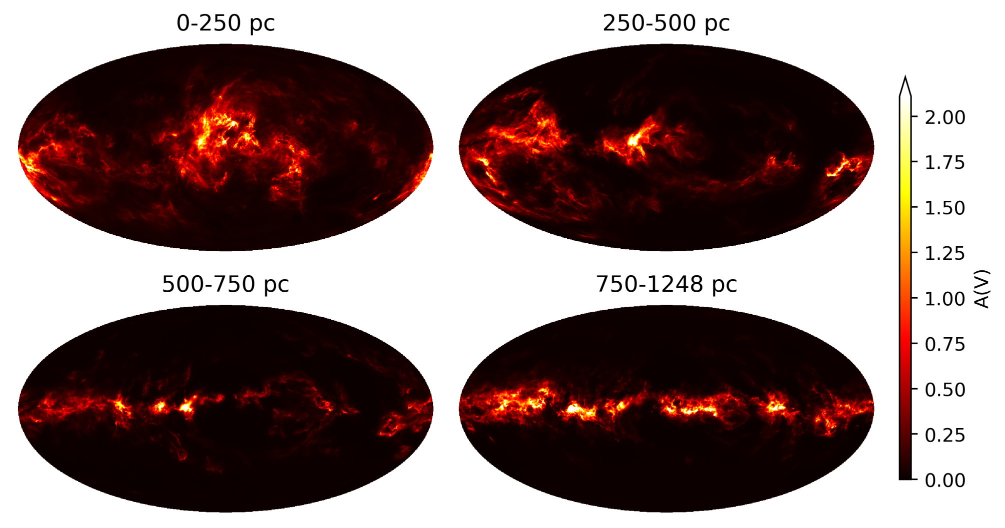
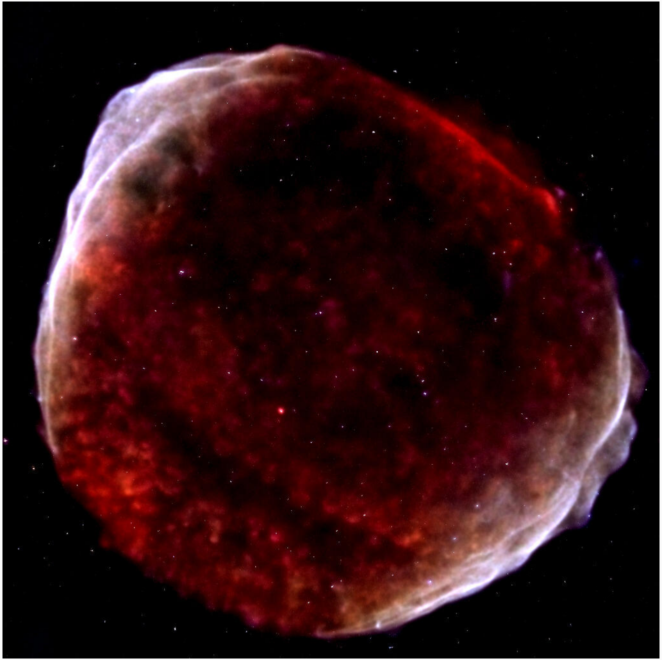
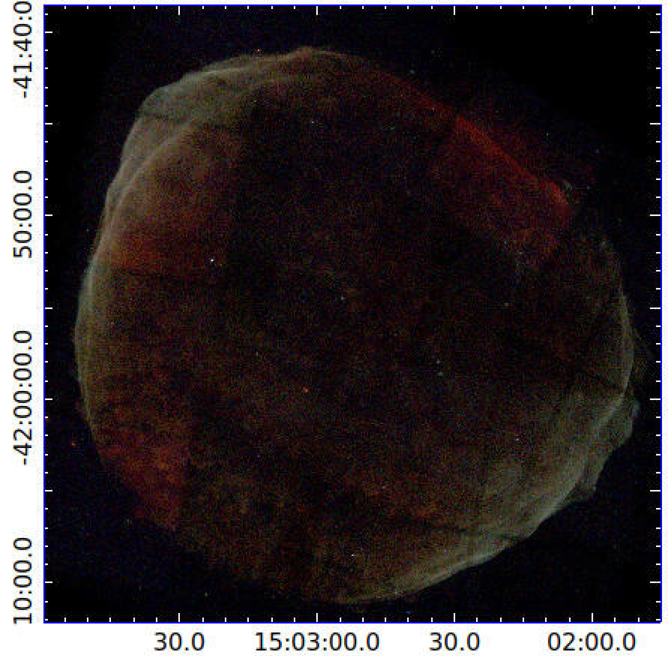


Chandra X-ray imaging [WEG+23]



Chandra X-ray imaging [WEG+23]



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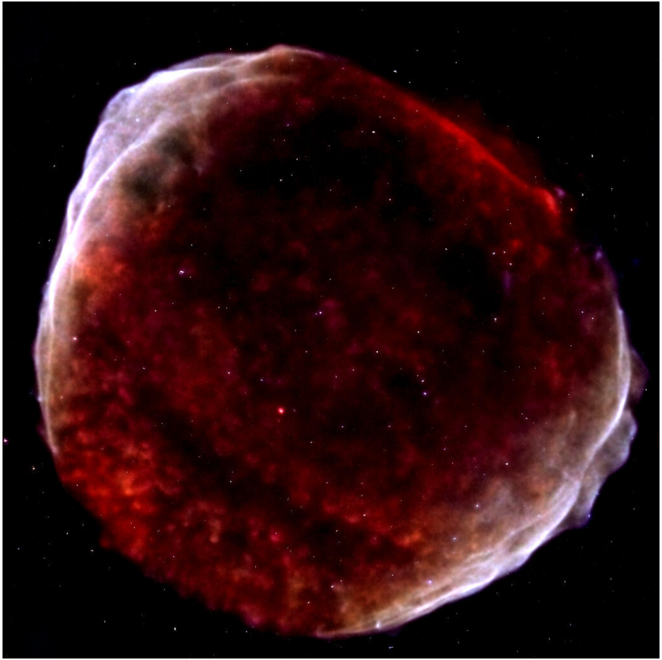

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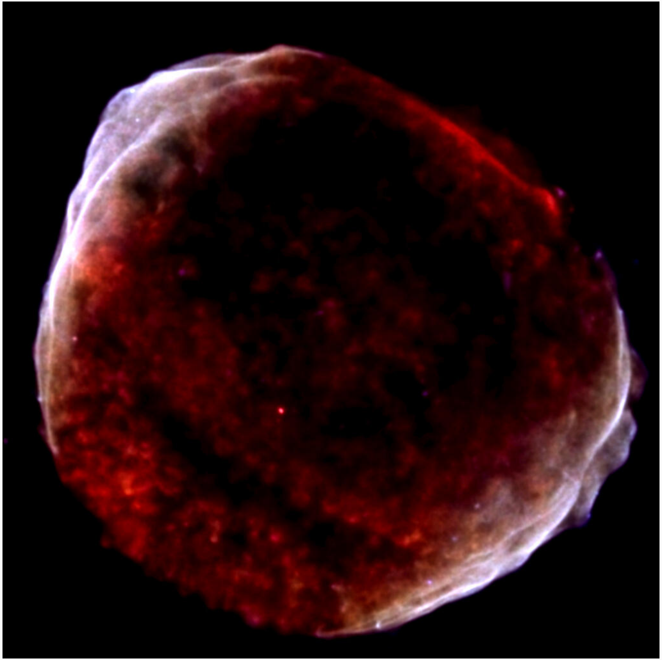

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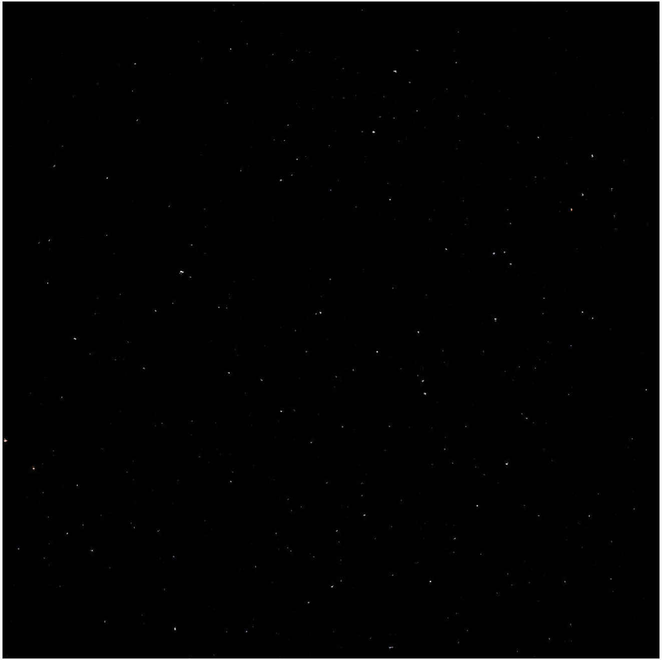

Fermi γ-ray sky

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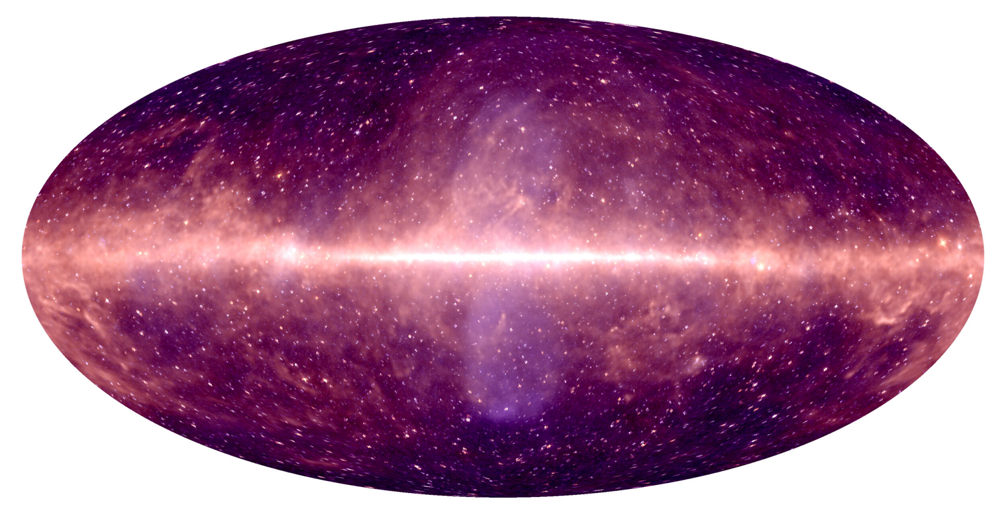

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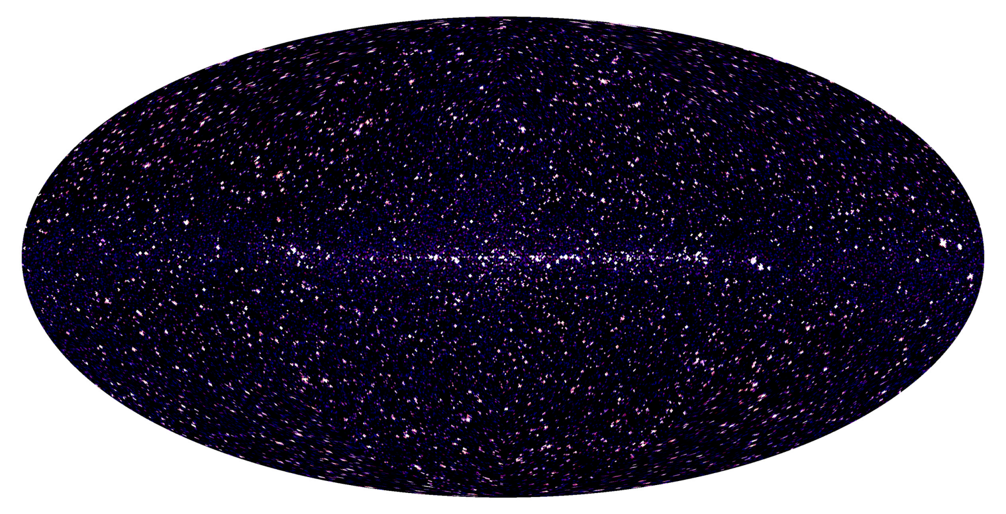

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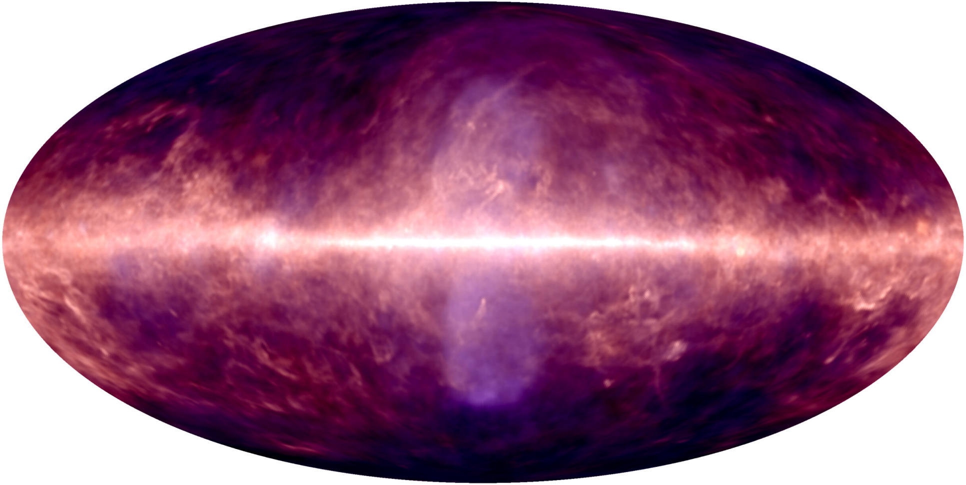
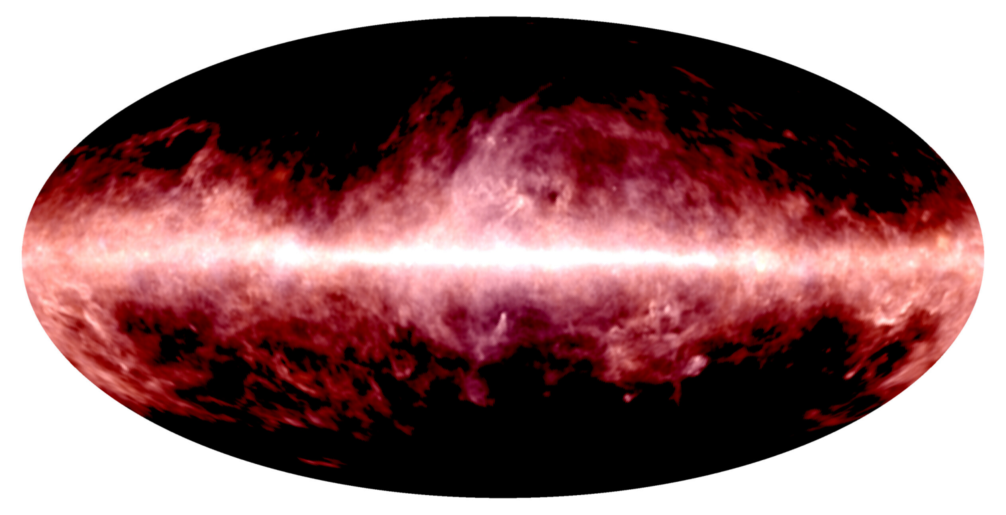


Fermi γ-ray sky



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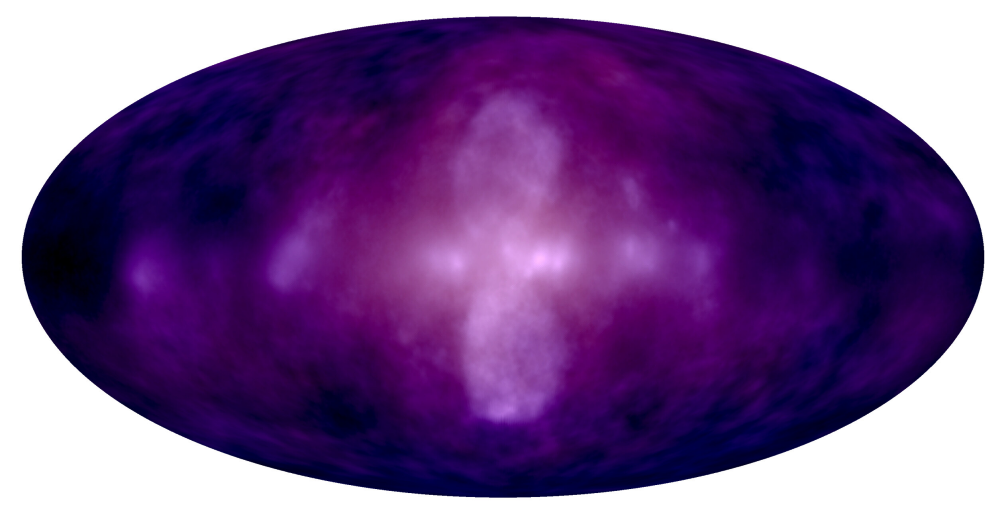

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Numerical Information Field Theory (NIFTy)

NIFTy

Code: https://gitlab.mpcdf.mpg.de/ift/nifty

Docs: https://ift.pages.mpcdf.de/nifty

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