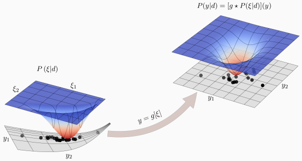
Recovering signals from astronomical data

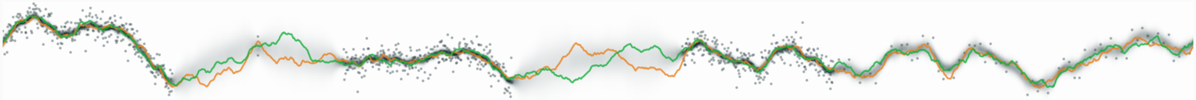


A probabilistic perspective

Philipp Frank[1]

Institute seminar: Max-Planck Institute for Astrophysics MPA, Garching, Germany, October 2, 2023

(1) Max-Planck Institute for Astrophysics MPA, Garching, Germany



Signal reconstruction for fields - Overview

- Why...
  - ... probabilistic estimators?
  - ... Bayes theorem?

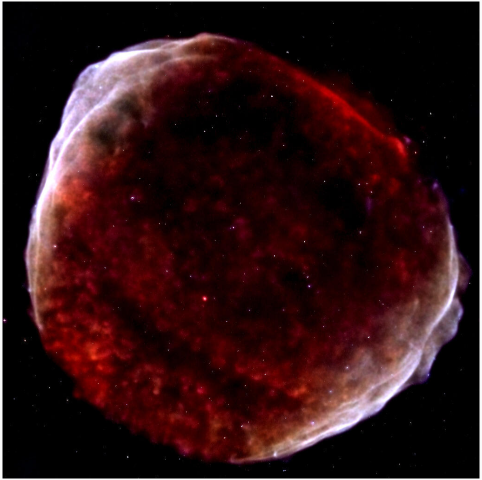


Figure 1: SN1006 from Chandra data[1]

[1] Westerkamp, Eberle, Guardiani, et al. 2023.

Signal reconstruction for fields - Overview

- Why...
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  - ... Bayes theorem?
- How do we...
  - ... model Priors & Likelihoods?
  - ... do (approximate) inference?

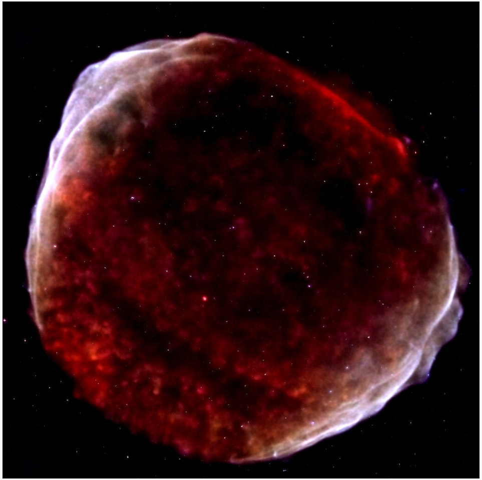

Figure 1: SN1006 from Chandra data[1]

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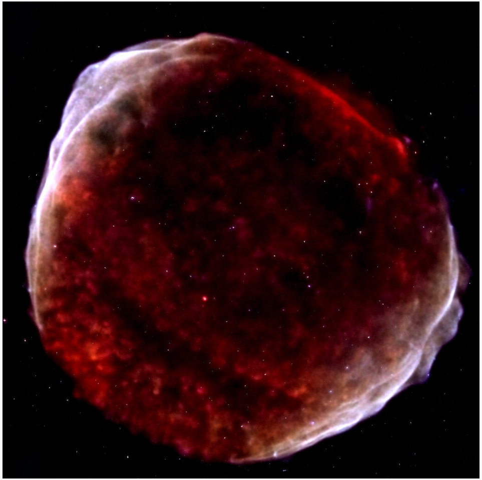
Signal reconstruction for fields - Overview

- Why...
  - ... probabilistic estimators?
  - ... Bayes theorem?
- How do we...
  - ... model Priors & Likelihoods?
  - ... do (approximate) inference?
- What?
  - Radio interferometry / VLBI
  - Faraday sky
  - GAIA 3D dust tomography
  - Chandra X-ray imaging
  - Fermi γ-ray sky



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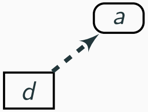
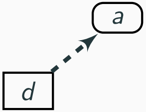
Probabilistic (Bayesian) Estimators



Given data d $\rightarrow$ obtain answers a about a system

Probabilistic (Bayesian) Estimators



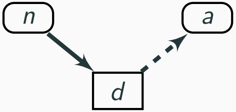
Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

$$\hat{a} = E\left(d; M\right) \quad .$$

With: $d =$ Data, $M =$ Model.

Probabilistic (Bayesian) Estimators



Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

$$\hat{a} = E\left(d; M\right) = \int a \, \mathcal{P}\left(a|d, M\right) \mathrm{d}a \quad .$$

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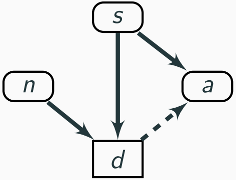
Probabilistic (Bayesian) Estimators



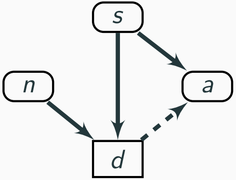
Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

$$\hat{a} = E\left(d; M\right) = \int a \, \mathcal{P}\left(a|d, M\right) \mathrm{d}a = \int a\left(s\right) \; \mathcal{P}\left(s|d, M\right) \mathrm{d}s \; .$$

With: $d =$ Data, $s =$ Signal, $M =$ Model.

Probabilistic (Bayesian) Estimators



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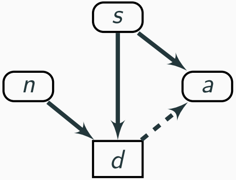
With: $d =$ Data, $s =$ Signal, $M =$ Model.

Product rule aka Bayes' Theorem

$$\mathcal{P}\left(s|d,M\right) = \frac{\mathcal{P}\left(s,d|M\right)}{\int \mathcal{P}\left(s,d|M\right)\ \mathrm{d}s} \quad .$$

Probabilistic (Bayesian) Estimators



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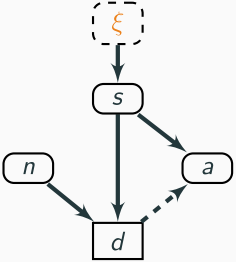
With: $d =$ Data, $s =$ Signal, $M =$ Model.

Product rule aka Bayes' Theorem

$$\mathcal{P}(s|d, M) = \frac{\mathcal{P}(s, d|M)}{\int \mathcal{P}(s, d|M)\ \mathrm{d}s} = \frac{\overbrace{\mathcal{P}(d|s\qquad, M)}^{\text{Likelihood}}\ \overbrace{\mathcal{P}(s|M)}^{\text{Prior}}}{\int \mathcal{P}(s, d|M)\ \mathrm{d}s}\ .$$

Probabilistic (Bayesian) Estimators



Given data d $\rightarrow$ obtain answers a about a system

Probabilistic estimator

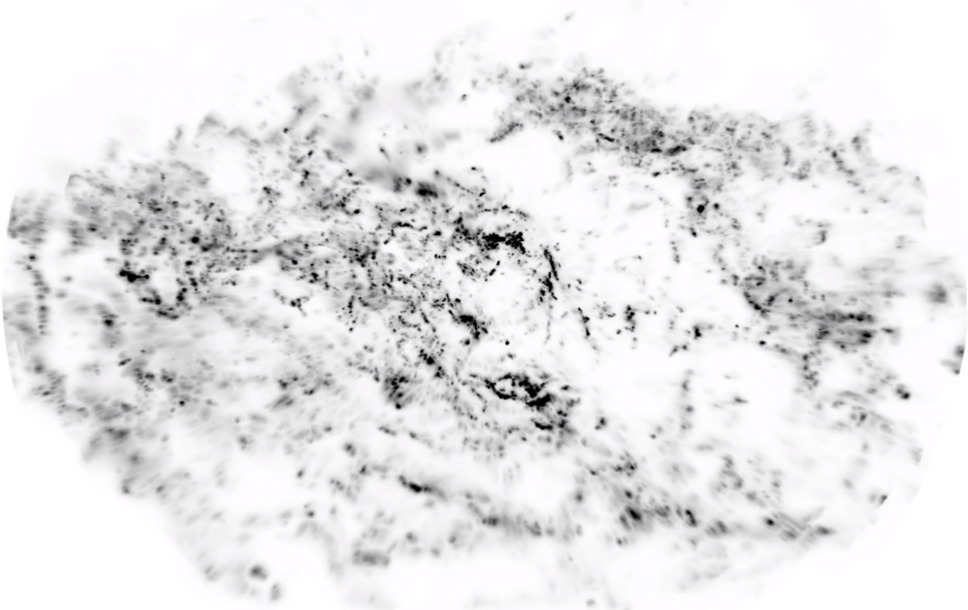
$$\hat{a} = E\left(d; M\right) = \int a\, \mathcal{P}\left(a|d, M\right) \mathrm{d}a = \int a\left(s\right)\ \mathcal{P}\left(s|d, M\right) \mathrm{d}s\ .$$

With: d = Data, s = Signal, M = Model.

Product rule aka Bayes' Theorem

$$\mathcal{P}\left(\xi|d, M\right) = \frac{\mathcal{P}\left(\xi, d|M\right)}{\int \mathcal{P}\left(\xi, d|M\right)\ \mathrm{d}\xi} = \frac{\overbrace{\mathcal{P}\left(d|s\left(\xi\right), M\right)}^{\text{Likelihood}}\ \overbrace{\mathcal{N}\left(\xi; 0, \mathbb{1}\right)}^{\text{Prior}}}{\int \mathcal{P}\left(\xi, d|M\right)\ \mathrm{d}\xi}\ .$$

With: ξ = Parameters.



Priors - Gaussian & generative processes

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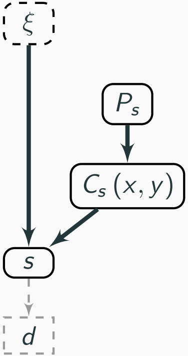

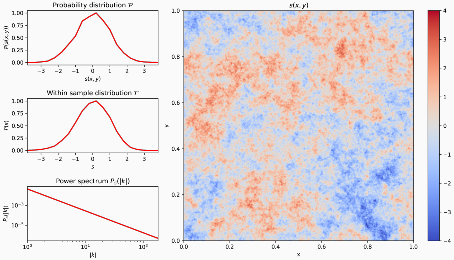

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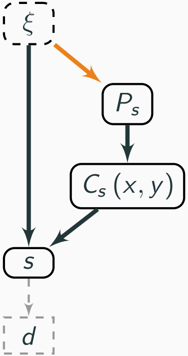
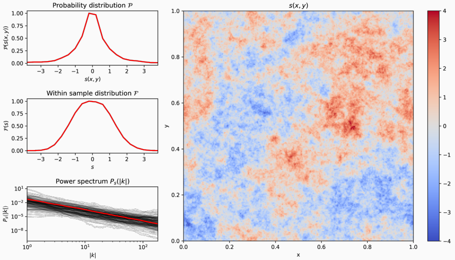
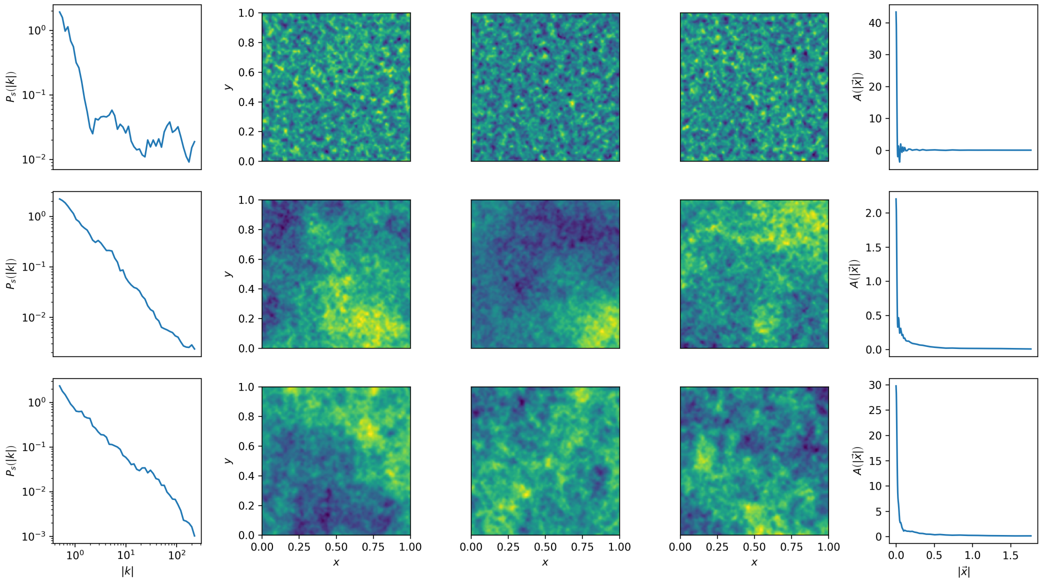
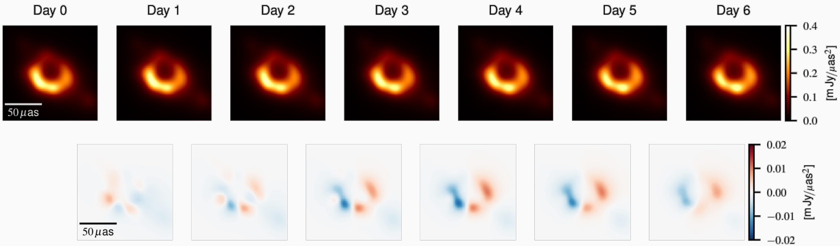


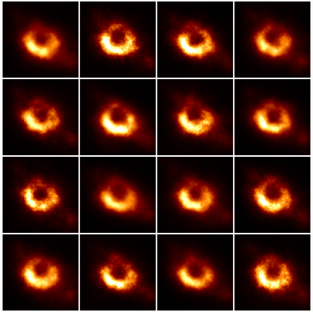


Priors - Gaussian & generative processes



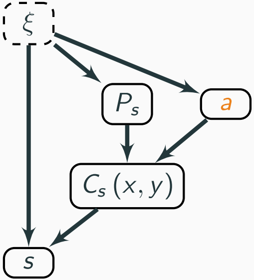
Priors - VLBI imaging of M87[2]

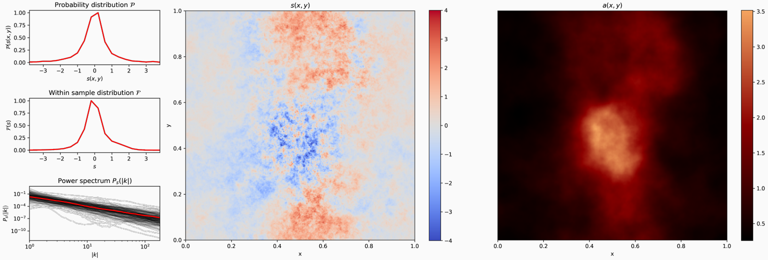




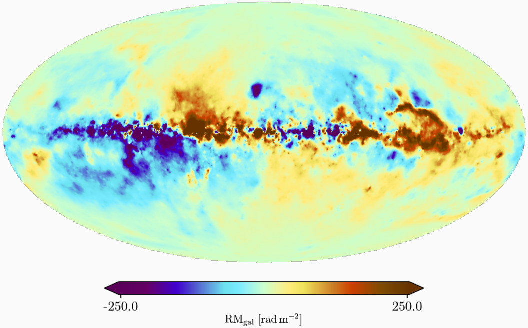
[2] Arras, Frank, Haim, et al. 2022.

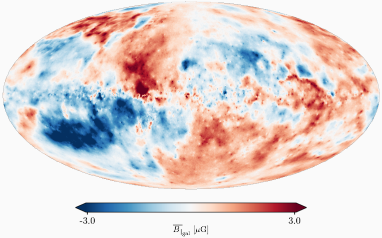
Priors - Gaussian & generative processes

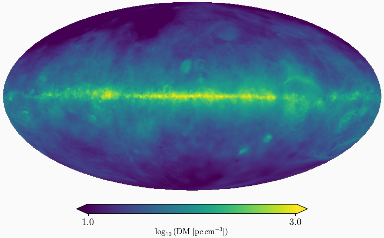




Priors - Faraday tomography[3]

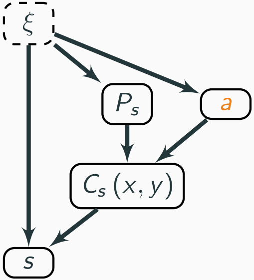


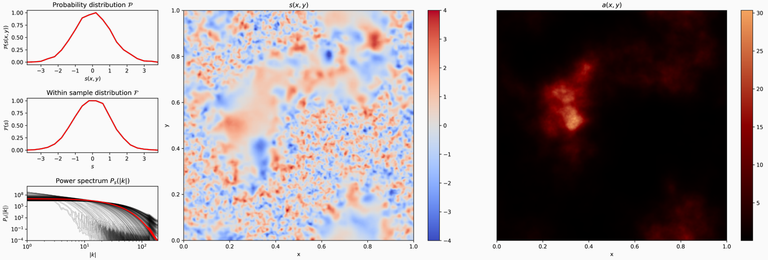




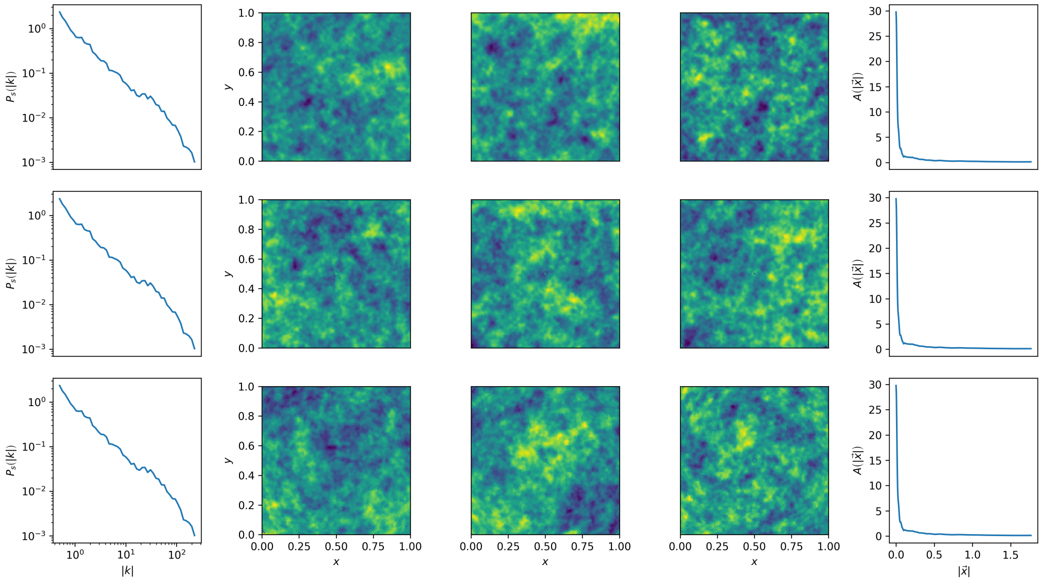
[3] Hutschenreuter, Haverkorn, Frank, et al. 2023.

Priors - Gaussian & generative processes





Priors - Gaussian & generative processes



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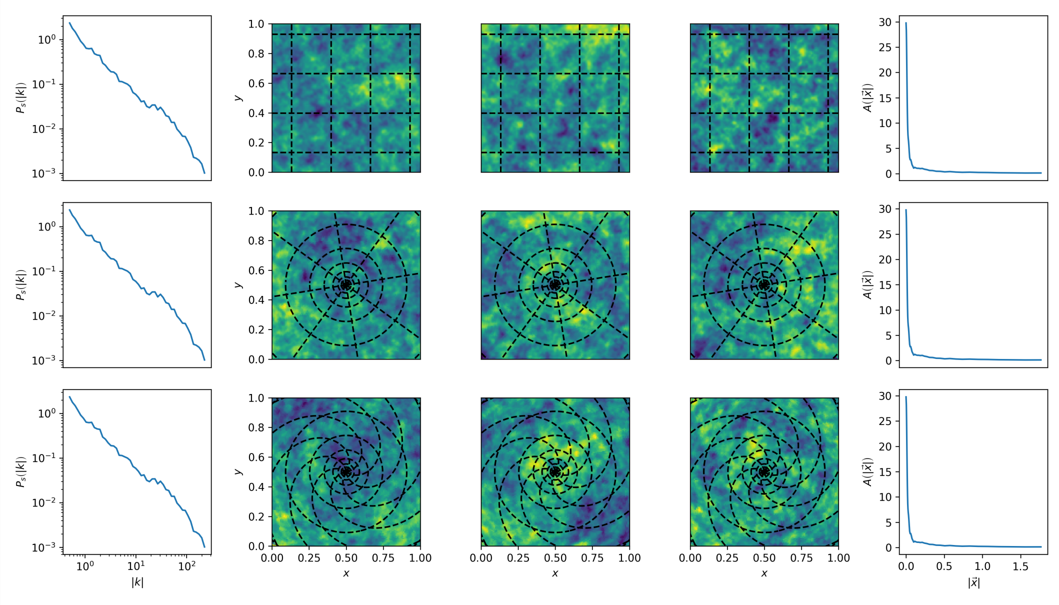
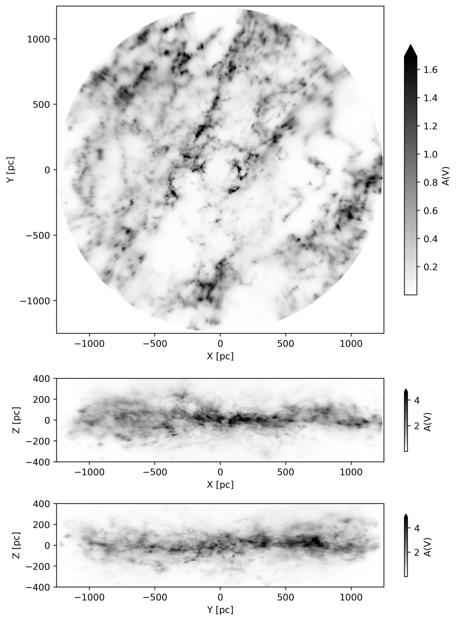


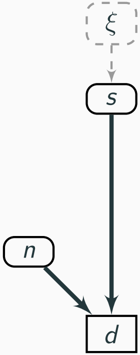
Priors - GAIA 3D dust tomography[4]

- HEALPix angular + Log-radial grid
- Log-Normal process on $\sim 0.66 \times 10^9$ voxels
- $\sim 80 \times 10^6$ measurements



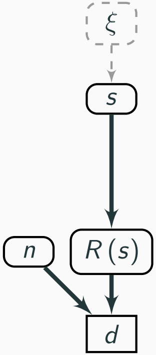
[4] Edenhofer, Zucker, Frank, et al. 2023.

Likelihoods - Instrument response



[5]Credit: https://cxc.harvard.edu/

Likelihoods - Instrument response



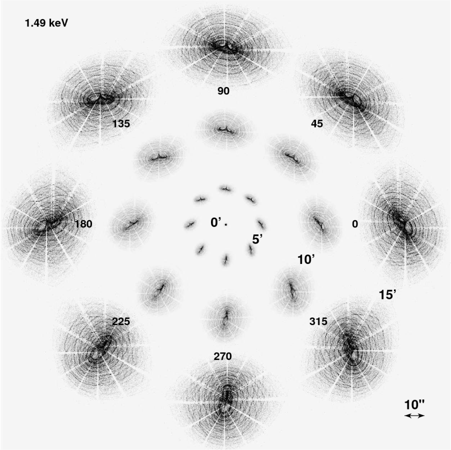
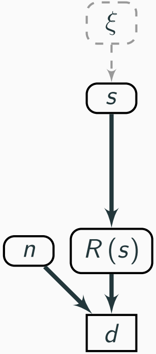


Figure 2: Simulated Chandra PSF[5]

[5] Credit: https://cxc.harvard.edu/

Likelihoods - Instrument response



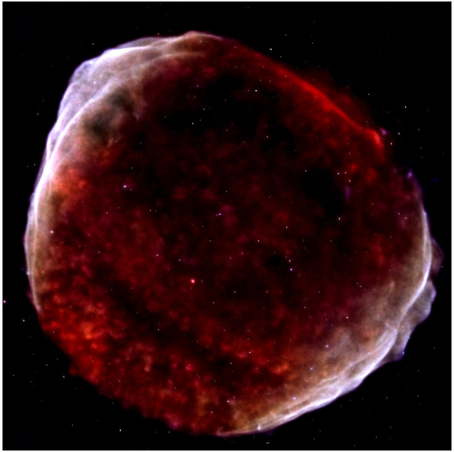
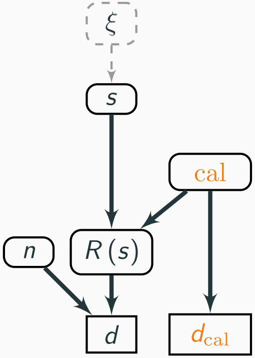


Figure 2: SN1006 from Chandra data[5]

[5] Westerkamp, Eberle, Guardiani, et al. 2023.



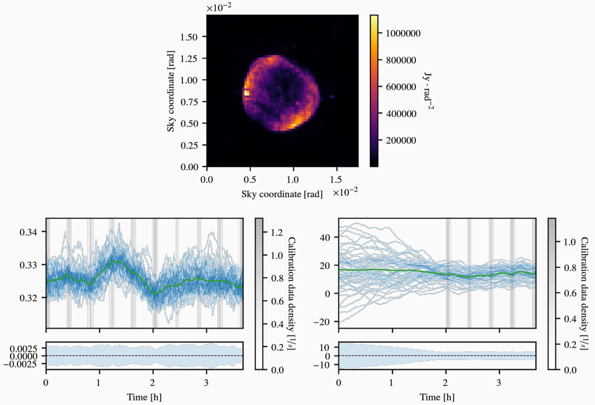
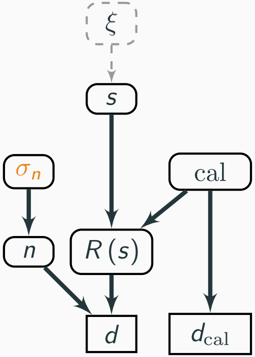


Figure 2: SN1006 from VLA data[5]

[5] Arras, Frank, Leike, et al. 2019.

Likelihoods - Instrument response



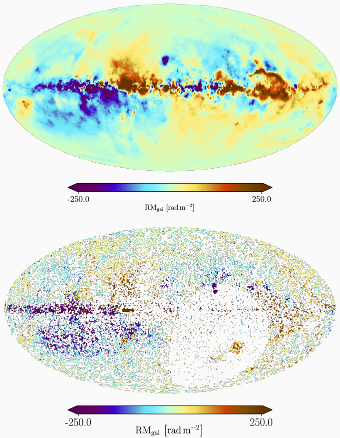
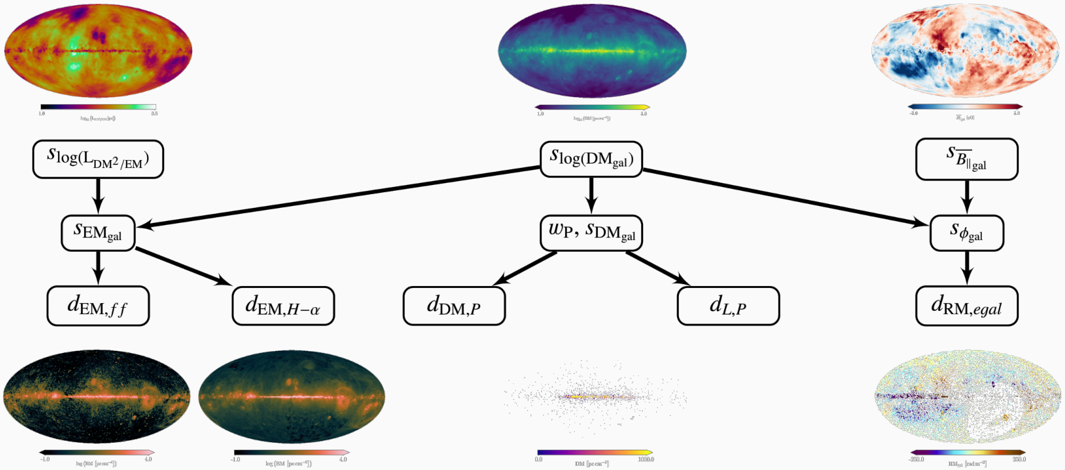


Figure 2: Faraday sky[5]

[5]Hutschenreuter, Haverkorn, Frank, et al. 2023.

Likelihoods - Faraday tomography[6]



[6]Hutschenreuter, Haverkorn, Frank, et al. 2023.

Approximate Inference

Posterior expectation

$$\langle f(\xi)\rangle_{\mathcal{P}(\xi|d)} = \int f \; \mathrm{d}\mathcal{P} = \int f(\xi) \; \mathcal{P}(\xi|d) \; \mathrm{d}\xi$$

Function: $f(\xi)$; Posterior: $\mathcal{P}(\xi|d)$; parameters: ξ; data: d.

Approximate Inference

Posterior expectation

$$\langle f(\xi) \rangle_{\mathcal{P}(\xi|d)} = \int f \; \mathrm{d}\mathcal{P} = \int f(\xi) \; \mathcal{P}(\xi|d) \; \mathrm{d}\xi$$

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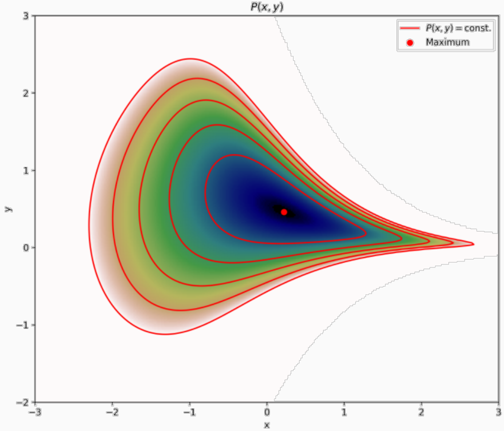
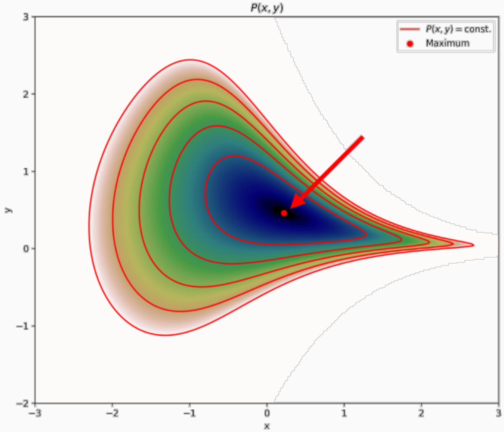


Approximate Inference

Posterior expectation

$$\langle f(\xi) \rangle_{\mathcal{P}(\xi|d)} = \int f \; \mathrm{d}\mathcal{P} = \int f(\xi) \; \mathcal{P}(\xi|d) \; \mathrm{d}\xi \approx f\left(\xi_{\mathrm{MAX}}\right)$$

Function: $f(\xi)$; Posterior: $\mathcal{P}(\xi|d)$; parameters: ξ; data: d.



Posterior expectation

$$\langle f(\xi)\rangle_{\mathcal{P}(\xi|d)} = \int f \, \mathrm{d}\mathcal{P} = \int f(\xi)\, \mathcal{P}(\xi|d)\, \mathrm{d}\xi \approx f\left(\xi_{\mathrm{MAX}}\right)$$

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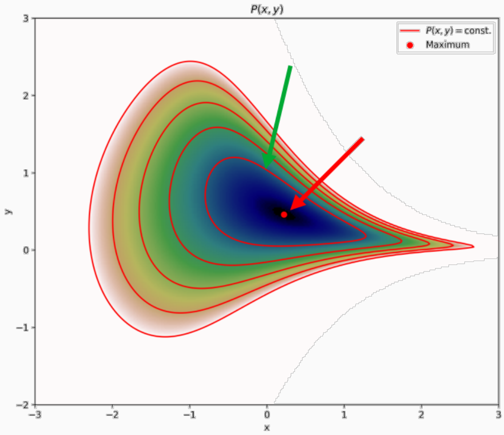
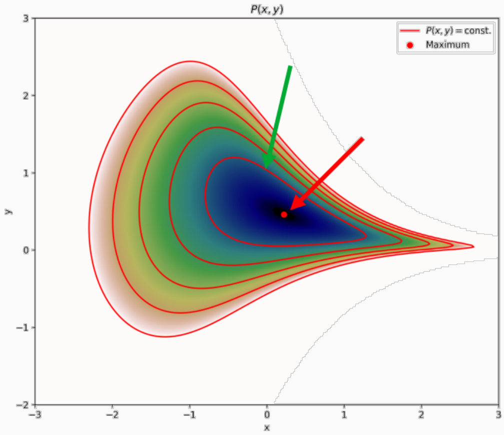


Posterior expectation

$$\langle f(\xi)\rangle_{\mathcal{P}(\xi|d)} = \int f \; \mathrm{d}\mathcal{P} = \int f(\xi)\; \mathcal{P}(\xi|d)\; \mathrm{d}\xi \approx \int f(\xi)\; \mathcal{Q}(\xi|d)\; \mathrm{d}\xi$$

Function: $f(\xi)$; Posterior: $\mathcal{P}(\xi|d)$; parameters: ξ; data: d.



Approximate Inference - Variational Inference (VI)

Kullback-Leibler divergence

$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = - \int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right)\ \mathcal{Q}_\sigma(\xi)\ \mathrm{d}\xi$$

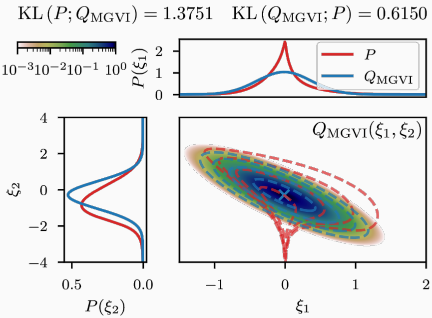
Posterior: $\mathcal{P}(\xi|d)$; Approximation: $\mathcal{Q}_\sigma(\xi)$; Variational parameters: σ.

[7] Knollmüller and Enßlin 2019.
[8] Frank, Leike, and Enßlin 2021.

Kullback-Leibler divergence

$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = -\int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right) \mathcal{Q}_\sigma(\xi)\, \mathrm{d}\xi$$

Posterior: $\mathcal{P}(\xi|d)$; Approximation: $\mathcal{Q}_\sigma(\xi)$; Variational parameters: σ.



[7]

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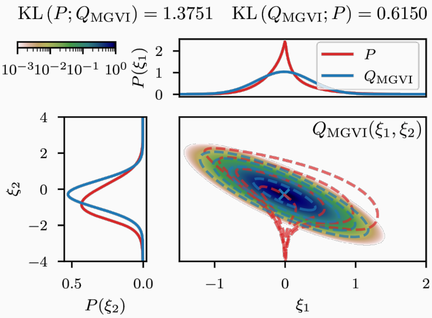
[7] Knollmüller and Enßlin 2019.
[8] Frank, Leike, and Enßlin 2021.

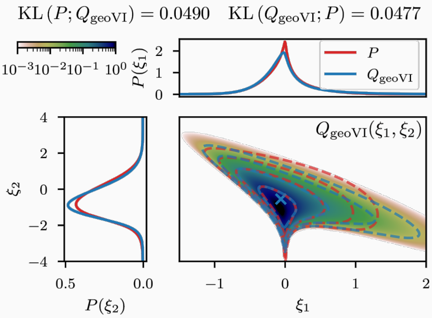
Kullback-Leibler divergence

$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = -\int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right)\, \mathcal{Q}_\sigma(\xi)\, \mathrm{d}\xi$$

Posterior: $\mathcal{P}(\xi|d)$; Approximation: $\mathcal{Q}_\sigma(\xi)$; Variational parameters: σ.



[7]

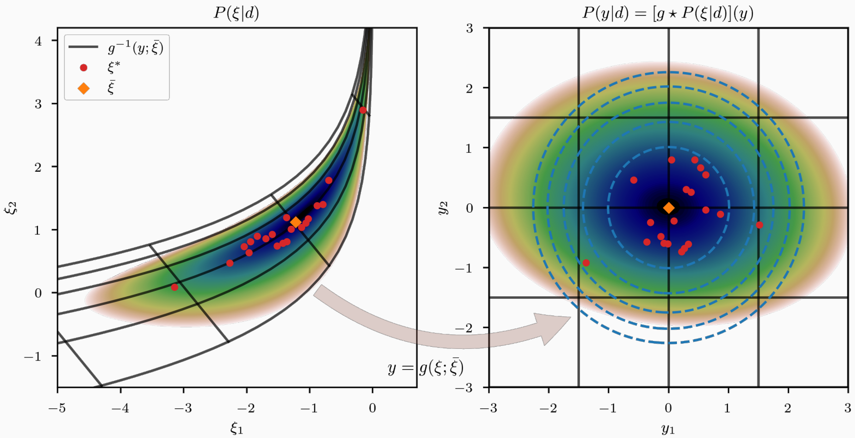


[8]

[7] Knollmüller and Enßlin 2019.
[8] Frank, Leike, and Enßlin 2021.

Geometric Variational Inference

Normalizing coordinate transformation $y = g_\sigma(\xi)$ with $\sigma = \bar{\xi}$.
Approximate distribution $\mathcal{Q}(y) = \mathcal{N}(y; 0, \mathbb{1})$



Conclusion - Remarks

- Estimators are model dependent
- Accurate instrument models crucial for inference

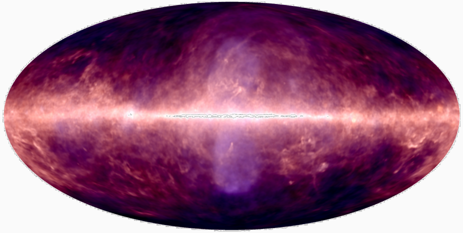


Figure 3: γ-ray sky from Fermi data[9]

[9]Platz, Knollmüller, Arras, et al. 2022.

Conclusion - Remarks

- Estimators are model dependent
- Accurate instrument models crucial for inference
- For fields:
  - GPs: Fast and flexible way to summarize prior knowledge...
  - (geo)VI: Scalable approximate inference...

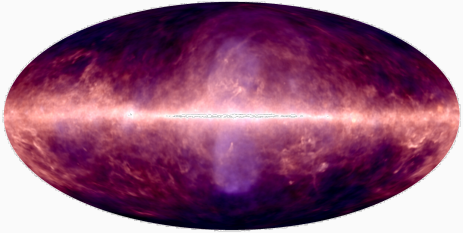


Figure 3: γ-ray sky from Fermi data[9]

[9] Platz, Knollmüller, Arras, et al. 2022.

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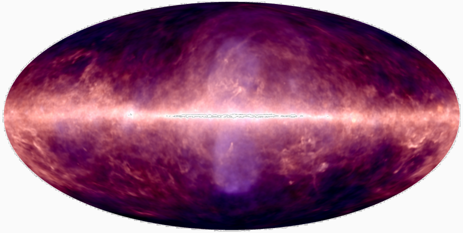

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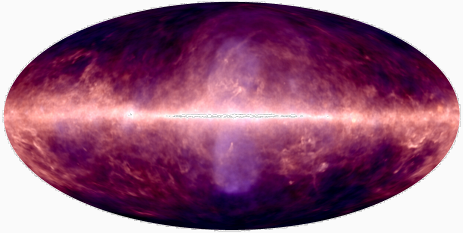

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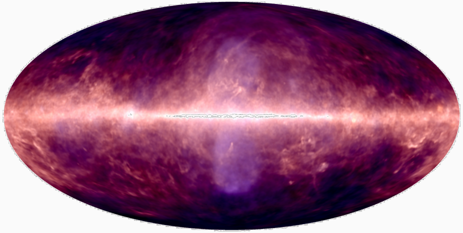

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