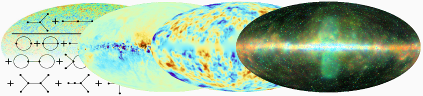
NIFTy



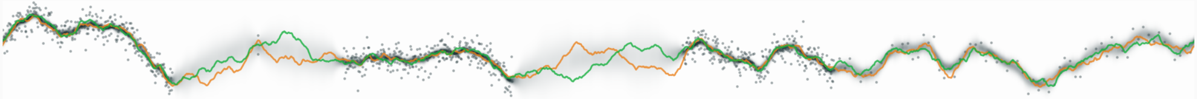
Numerical information field theory

Bayesian imaging using IFT

Philipp Frank[1]

VLTI and ALMA Synthesis Imaging Workshop - ESO Garching, January 11, 2023

(1) Max-Planck Institute for Astrophysics, Garching, Germany



Overview

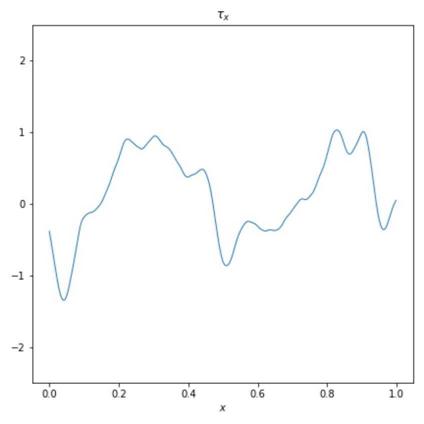
Code: **https://gitlab.mpcdf.mpg.de/ift/nifty**

NIFTy [ABE$^+$19]:

- python library for statistical inference
- differentiable generative models
- flexible gaussian processes (correlated field model)
- variety of observational likelihoods
- variational inference

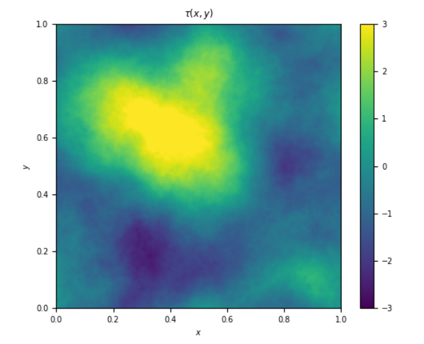
Spaces

NIFTy - Spaces



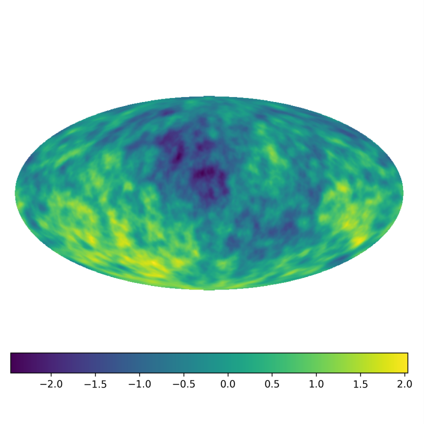
```python
import nifty8 as ift

# 1-dimensional regular grid space
# with 128 pixels and pixelsize 1/128
space = ift.RGSpace(128, 1/128)

...
```

NIFTy - Spaces



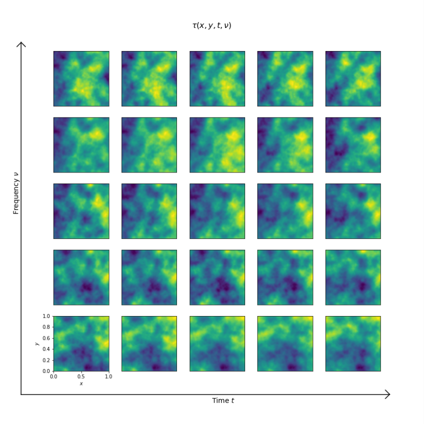
```python
import nifty8 as ift

# 2-dimensional regular grid space
# with 128x128 pixels and pixelsizes 1/128
space = ift.RGSpace((128, 128), (1/128, 1/128))

...
```

NIFTy - Spaces



```python
import nifty8 as ift

# 2-dimensional spherical (HEALPiX) space
# with nside 128
space = ift.HPSpace(128)

...
```

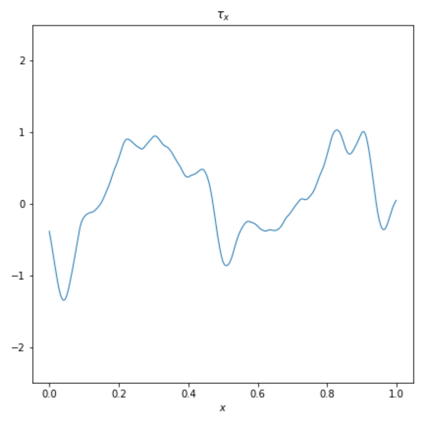
NIFTy - Spaces



```python
import nifty8 as ift

# 2-dimensional regular grid space
# with 128x128 pixels
image_dom = ift.RGSpace((128, 128))

# frequency & time domain with
# 5 regularly spaced pixels
time = ift.RGSpace(5)
freq = ift.RGSpace(5)

# Set up joint space
space = ift.makeDomain((freq, time, image_dom))

...
```

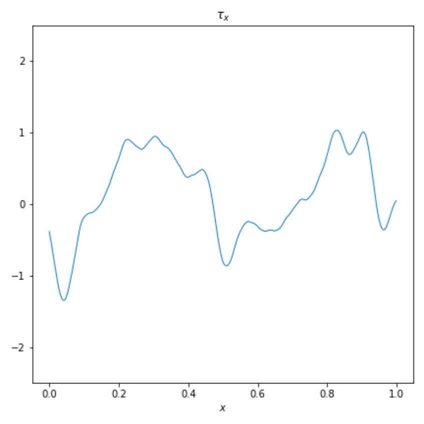
CorrelatedFields

NIFTy - CorrelatedFields [AFH+22]



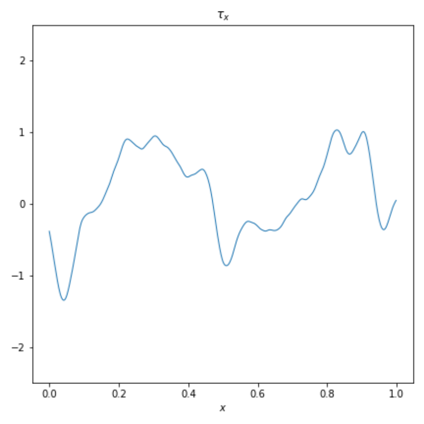
```python
import nifty8 as ift

# 1-dimensional regular grid space
# with 128 pixels and pixelsize 1/128
space = ift.RGSpace(128, 1/128)












... # Plot 'tau'
```

NIFTy - CorrelatedFields [AFH+22]



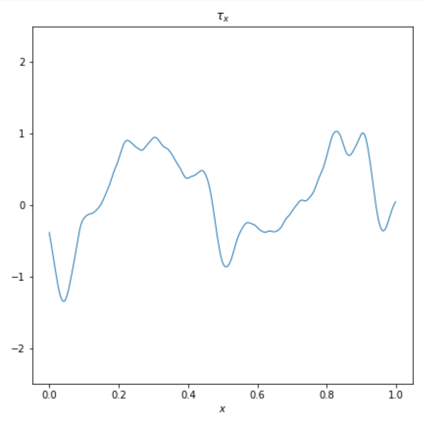
```python
import nifty8 as ift

# 1-dimensional regular grid space
# with 128 pixels and pixelsize 1/128
space = ift.RGSpace(128, 1/128)

# Define a Gaussian random processes on 'space'
args = {...} # Hyperparameters for GP model
model = ift.SimpleCorrelatedField(space, **args)








... # Plot 'tau'
```

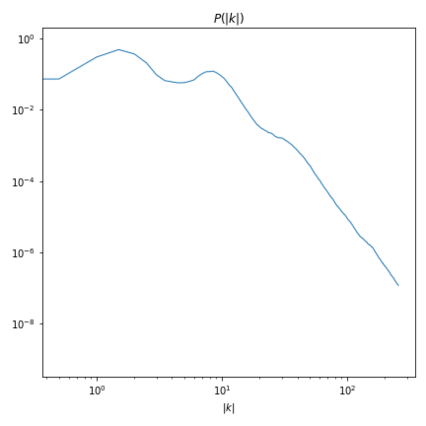
NIFTy - CorrelatedFields [AFH+22]



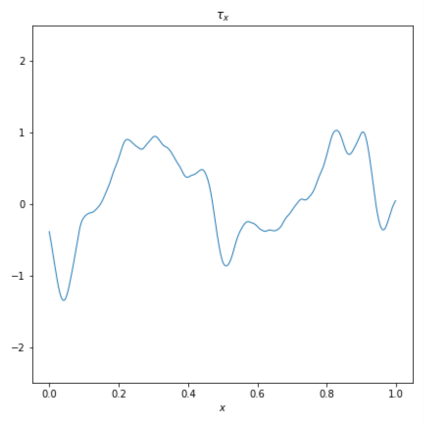
```python
import nifty8 as ift

# 1-dimensional regular grid space
# with 128 pixels and pixelsize 1/128
space = ift.RGSpace(128, 1/128)

# Define a Gaussian random processes on 'space'
args = {...} # Hyperparameters for GP model
model = ift.SimpleCorrelatedField(space, **args)

# Draw a random realization of standard normal
# distributed variables
realization = ift.from_random(model.domain)
# Apply model to get a realization
tau = model(realization)

... # Plot 'tau'
```

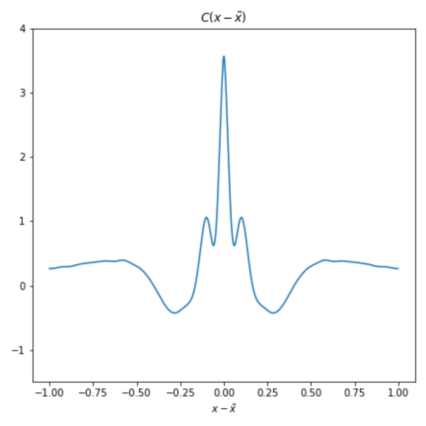
NIFTy - CorrelatedFields [AFH+22]



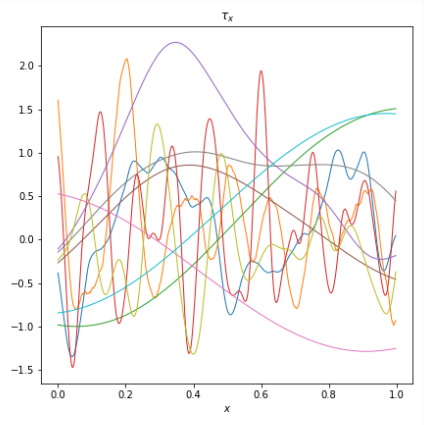


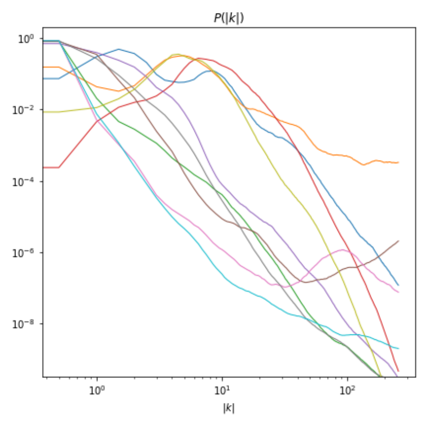
NIFTy - CorrelatedFields [AFH+22]



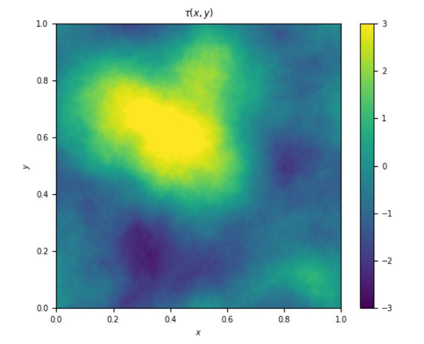


NIFTy - CorrelatedFields [AFH^{+}22]





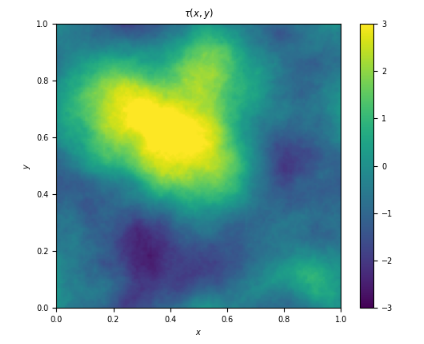
NIFTy - CorrelatedFields [AFH+22]



```python
import nifty8 as ift

# 2-dimensional regular grid space
# with 128x128 pixels and pixelsizes 1/128
space = ift.RGSpace((128, 128), (1/128, 1/128))

# Define a Gaussian random processes on 'space'
args = {...} # Hyperparameters for GP model
model = ift.SimpleCorrelatedField(space, **args)

# Draw a random realization of standard normal
# distributed variables
realization = ift.from_random(model.domain)
# Apply model to get a model realization
tau = model(realization)

... # Plot 'tau'
```

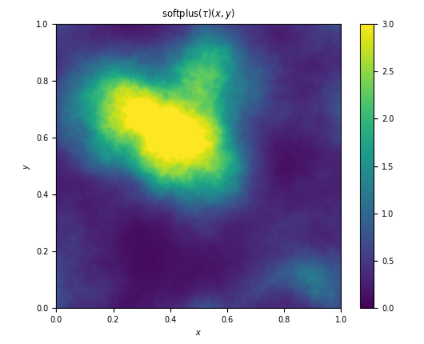
Data model & Variational inference

NIFTy - Variational Inference [FLE21]



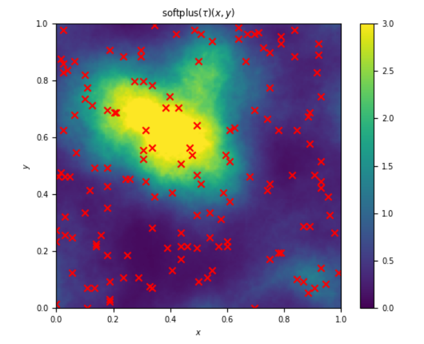
```
model = ... # model for tau
















...
```

NIFTy - Variational Inference [FLE21]



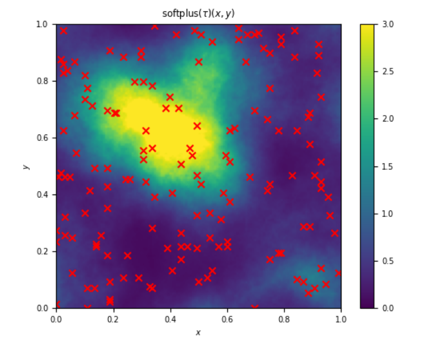
```
model = ift.softplus(model) # apply nonlinearity















...
```

NIFTy - Variational Inference [FLE21]



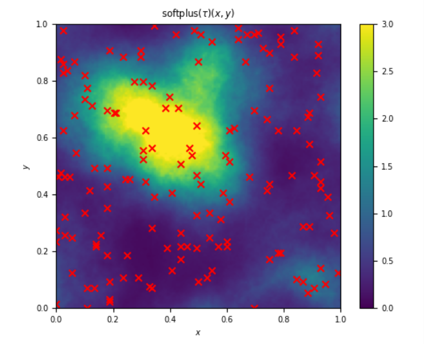
```
model = ift.softplus(model) # apply nonlinearity

# Set up a random response
flags = np.random.binomial(1, 0.99, size = 128**2)
flags = ift.makeField(space, flags)
Response = ift.MaskOperator(flags)











...
```

NIFTy - Variational Inference [FLE21]



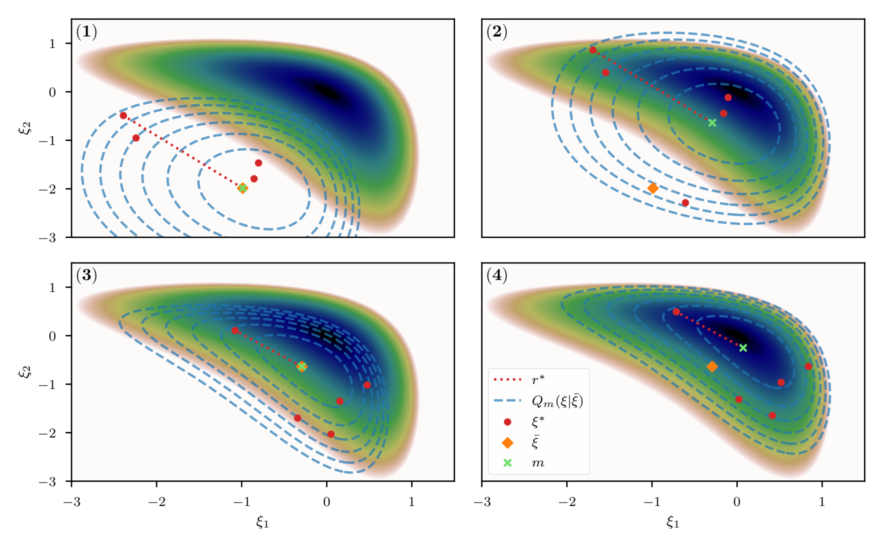
```
model = ift.softplus(model) # apply nonlinearity

# Set up a random response
flags = np.random.binomial(1, 0.99, size = 128**2)
flags = ift.makeField(space, flags)
Response = ift.MaskOperator(flags)

# Define observational model and likelihood
data, noise_icov = # load data and noise
lh = ift.GaussianEnergy(data = data,
            inverse_covariance = noise_icov)
likelihood = lh @ Response(model)





...
```

NIFTy - Variational Inference [FLE21]



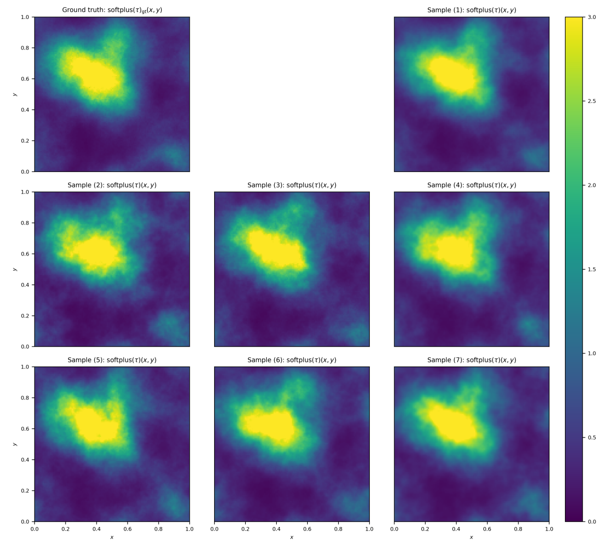
```
model = ift.softplus(model) # apply nonlinearity

# Set up a random response
flags = np.random.binomial(1, 0.99, size = 128**2)
flags = ift.makeField(space, flags)
Response = ift.MaskOperator(flags)

# Define observational model and likelihood
data, noise_icov = # load data and noise
lh = ift.GaussianEnergy(data = data,
            inverse_covariance = noise_icov)
likelihood = lh @ Response(model)

# Generate approximate posterior samples
# using variational inference (geoVI)
samples = ift.optimize_kl(lh, **params)
...
```

Geometric Variational Inference (geoVI) [FLE21]

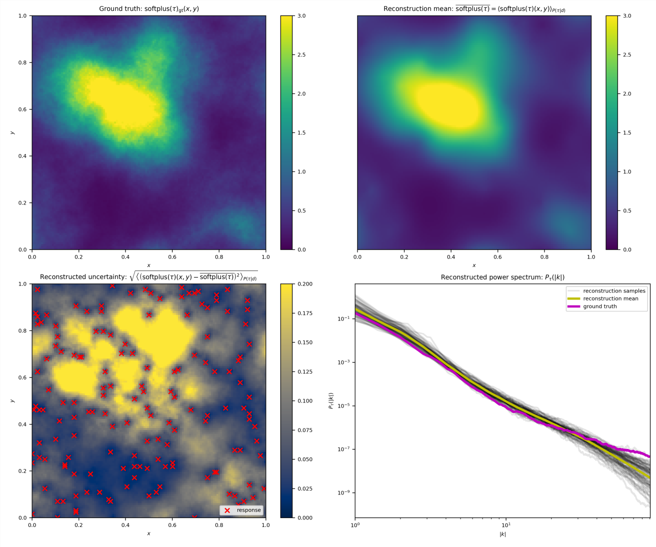


→ not 2- but (# of pixels + # spectrum parameters)-dimensional probability distributions!

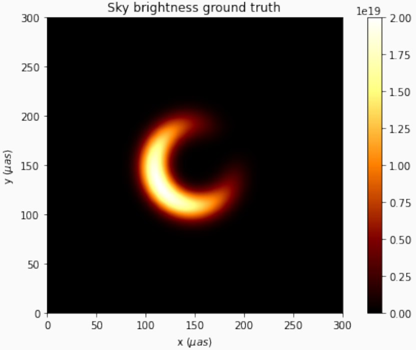
NIFTy - Variational Inference [FLE21]



NIFTy - Variational Inference [FLE21]

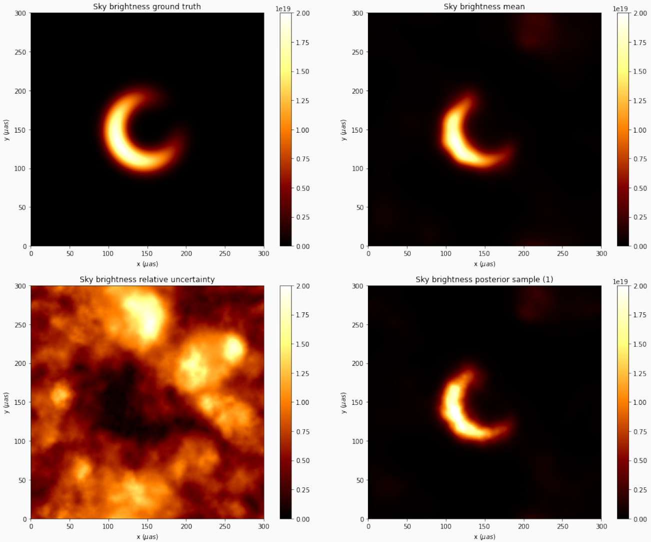


Interferometric imaging

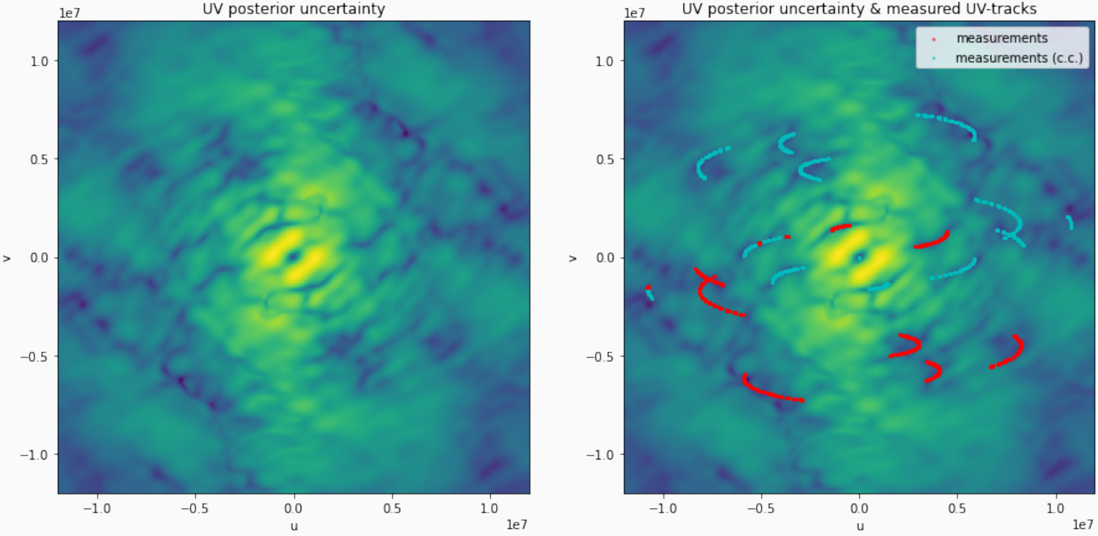


```
model = ift.exp(model) # apply nonlinearity

# Set up a vlbi response
import resolve as rve
observation = rve.Observation.load('data path...')
Response = rve.InterferometryResponse(
                                observation, model.target)

# Define observational model and likelihood
data, noise_icov = # load data and noise
lh = ift.GaussianEnergy(data = data,
             inverse_covariance = noise_icov)
likelihood = lh @ Response(model)




...
```

Interferometric imaging



Interferometric imaging



NIFTy - JAX

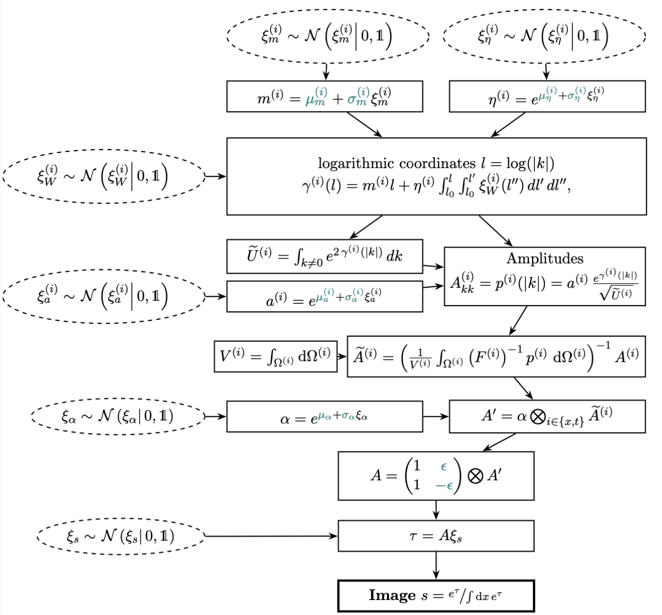
```python
import jax
import nifty8.re as jft

model = # some jifty model ...





model = jax.jit(model) # just-in-time compilation


# ... run on GPU/TPU




...
```

References

- Philipp Arras, Mihai Baltac, Torsten A Ensslin, Philipp Frank, Sebastian Hutschenreuter, Jakob Knollmueller, Reimar Leike, Max-Niklas Newrzella, Lukas Platz, Martin Reinecke, et al.
  Nifty5: Numerical information field theory v5.
  Astrophysics Source Code Library, pages ascl–1903, 2019.
- Philipp Arras, Philipp Frank, Philipp Haim, Jakob Knollmüller, Reimar Leike, Martin Reinecke, and Torsten A. Enßlin.
  Variable structures in m87* from space, time and frequency resolved interferometry.
  Nature Astronomy, 6(2):259–269, 2022.
- Philipp Frank, Reimar Leike, and Torsten A. Enßlin.
  Geometric variational inference.
  Entropy, 23(7), 2021.

Backup

Backup



$\xi_m^{(i)} \sim \mathcal{N}\left(\xi_m^{(i)} \middle| 0, \mathbb{1}\right)$

$\xi_\eta^{(i)} \sim \mathcal{N}\left(\xi_\eta^{(i)} \middle| 0, \mathbb{1}\right)$

$m^{(i)} = \mu_m^{(i)} + \sigma_m^{(i)} \xi_m^{(i)}$

$\eta^{(i)} = e^{\mu_\eta^{(i)} + \sigma_\eta^{(i)} \xi_\eta^{(i)}}$

$\xi_W^{(i)} \sim \mathcal{N}\left(\xi_W^{(i)} \middle| 0, \mathbb{1}\right)$

logarithmic coordinates $l = \log(|k|)$

$\gamma^{(i)}(l) = m^{(i)} l + \eta^{(i)} \int_{l_0}^{l} \int_{l_0}^{l'} \xi_W^{(i)}(l'')\, dl'\, dl''$,

$\widetilde{U}^{(i)} = \int_{k \neq 0} e^{2\,\gamma^{(i)}(|k|)}\, dk$

$\xi_a^{(i)} \sim \mathcal{N}\left(\xi_a^{(i)} \middle| 0, \mathbb{1}\right)$

$a^{(i)} = e^{\mu_a^{(i)} + \sigma_a^{(i)} \xi_a^{(i)}}$

Amplitudes

$A_{kk}^{(i)} = p^{(i)}(|k|) = a^{(i)} \frac{e^{\gamma^{(i)}(|k|)}}{\sqrt{\widetilde{U}^{(i)}}}$

$V^{(i)} = \int_{\Omega^{(i)}} \mathrm{d}\Omega^{(i)}$

$\widetilde{A}^{(i)} = \left(\frac{1}{V^{(i)}} \int_{\Omega^{(i)}} \left(F^{(i)}\right)^{-1} p^{(i)}\ \mathrm{d}\Omega^{(i)} \right)^{-1} A^{(i)}$

$\xi_\alpha \sim \mathcal{N}\left(\xi_\alpha | 0, \mathbb{1}\right)$

$\alpha = e^{\mu_\alpha + \sigma_\alpha \xi_\alpha}$

$A' = \alpha \bigotimes_{i \in \{x,t\}} \widetilde{A}^{(i)}$

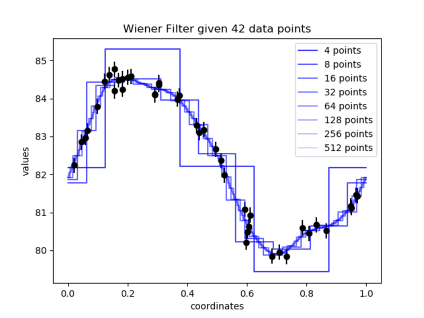
$A = \begin{pmatrix} 1 & \epsilon \\ 1 & -\epsilon \end{pmatrix} \bigotimes A'$

$\xi_s \sim \mathcal{N}\left(\xi_s | 0, \mathbb{1}\right)$

$\tau = A \xi_s$

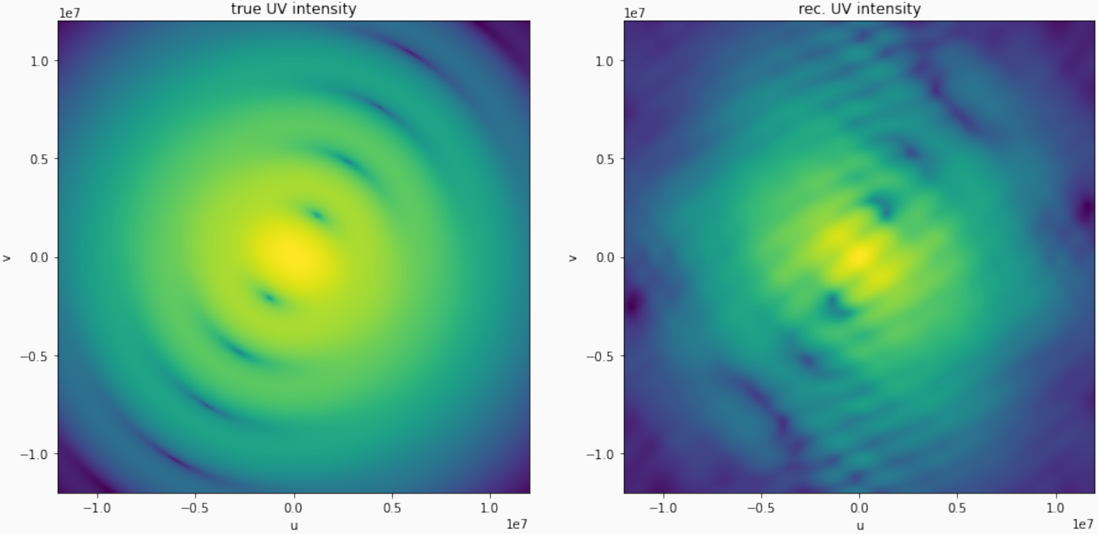
Image $s = {e^\tau}/{\int \mathrm{d}x\, e^\tau}$

Backup

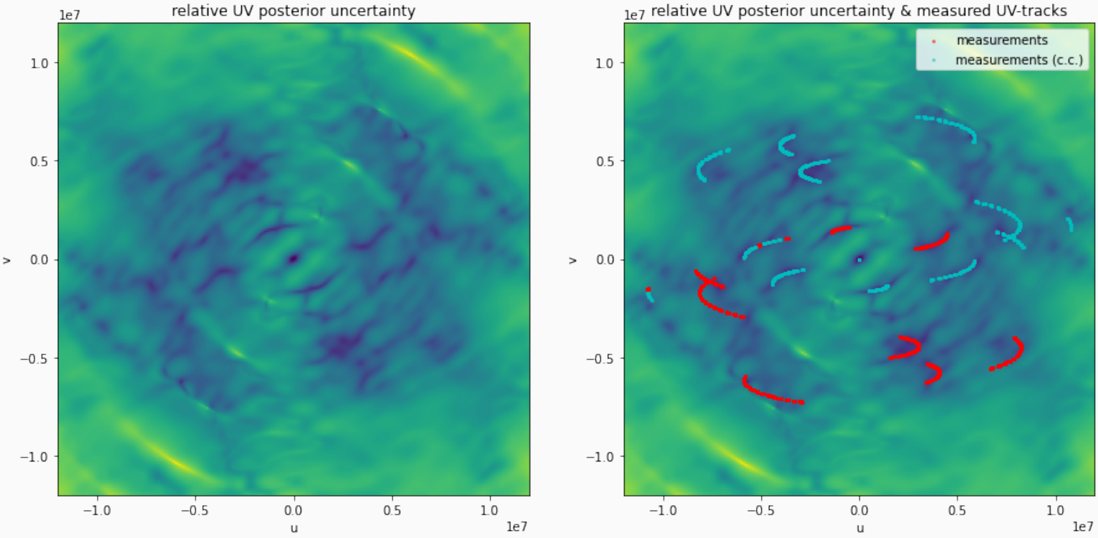


```python
import nifty8 as ift

for n in range(7):
        # 1-dimensional regular grid space
        # with 2^(n + 2) pixels
        space = ift.RGSpace(2**(n + 2))

        ...
```

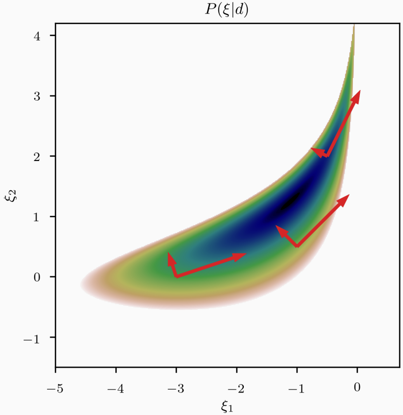
Interferometric imaging



Interferometric imaging



Geometric Variational Inference (geoVI) [?]

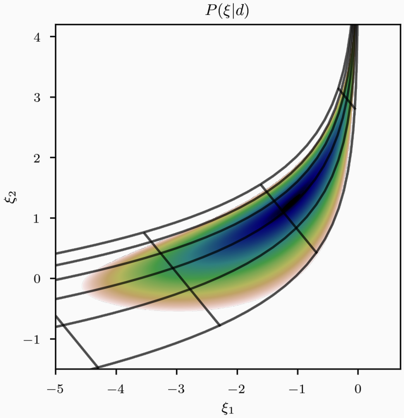


Information Hamiltonian $\mathcal{H}(\xi|d)$: $-\log\left(\mathcal{P}(\xi|d)\right)$

Posterior metric $\mathcal{M}(\xi)$: $\mathcal{M}_{\mathrm{lh}}(\xi) + \mathbb{1}$

Fisher information metric $\mathcal{M}_{\mathrm{lh}}(\xi)$: $\left\langle \frac{\partial^2 \mathcal{H}(d|\xi)}{\partial\xi\partial\xi'} \right\rangle_{\mathcal{P}(d|\xi)}$

Geometric Variational Inference (geoVI) [?]

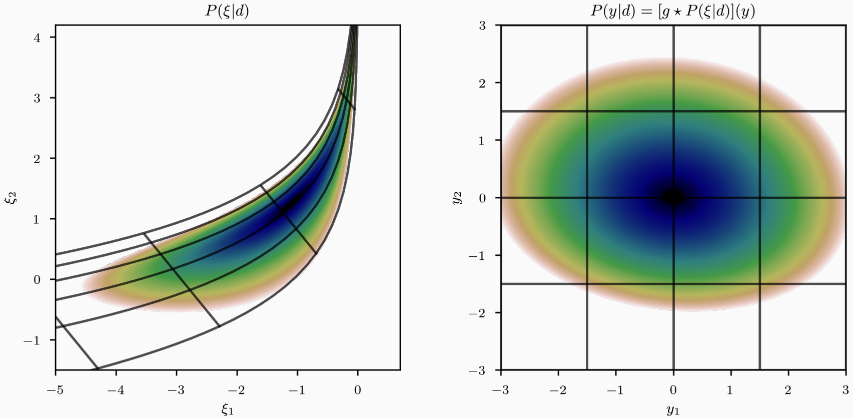


Information Hamiltonian $\mathcal{H}(\xi|d)$: $-\log\left(\mathcal{P}(\xi|d)\right)$

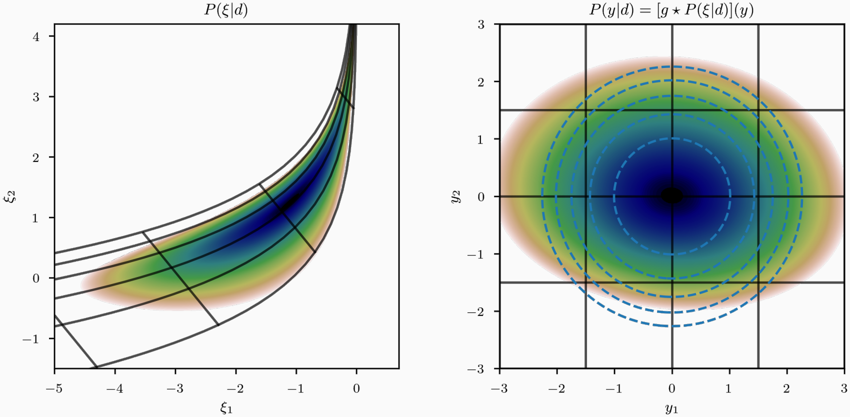
Posterior metric $\mathcal{M}(\xi)$: $\mathcal{M}_{\mathrm{lh}}(\xi) + \mathbb{1}$

Fisher information metric $\mathcal{M}_{\mathrm{lh}}(\xi)$: $\left\langle \frac{\partial^2 \mathcal{H}(d|\xi)}{\partial\xi\partial\xi'} \right\rangle_{\mathcal{P}(d|\xi)}$

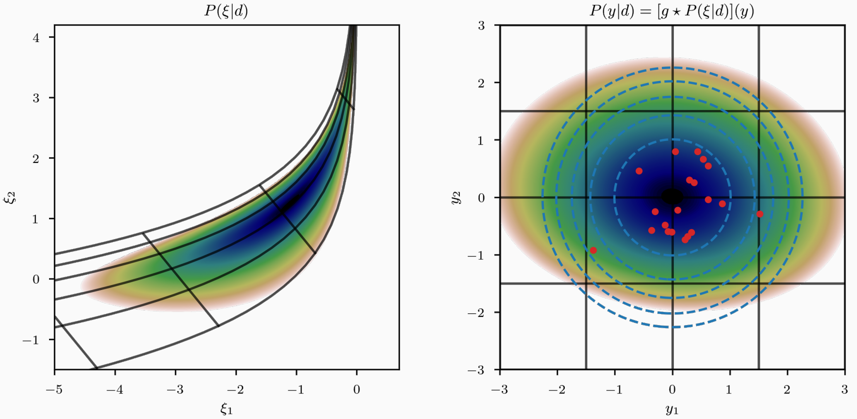
Geometric Variational Inference (geoVI) [?]



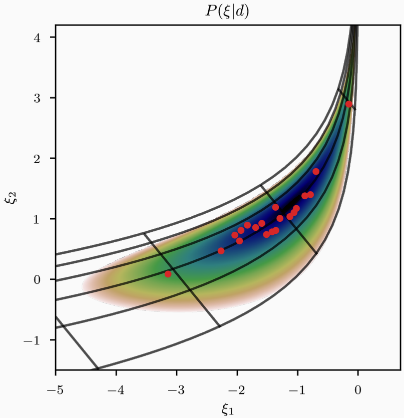
Geometric Variational Inference (geoVI) [?]



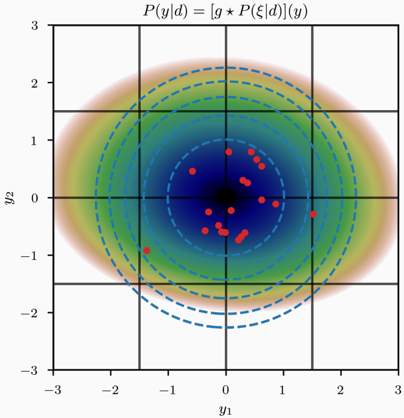
Geometric Variational Inference (geoVI) [?]



Geometric Variational Inference (geoVI) [?]



$P(\xi|d)$



$P(y|d) = [g \star P(\xi|d)](y)$