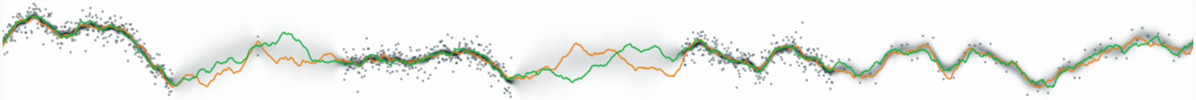
Geometric Variational Inference

Variational inference in information field theory

Philipp Frank[1], Torsten Ensslin[1,2]

MIT Haystack Observatory, Westford MA, USA; November 9, 2022

(1) Max-Planck Institute for Astrophysics MPA, Garching, Germany
(2) Ludwig-Maximilians University LMU, Munich, Germany



Variational Inference

Variational Inference (VI)

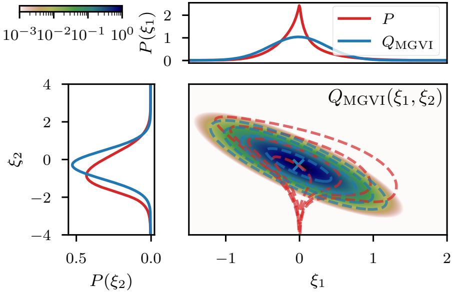
Kullback-Leibler divergence

$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = -\int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right) \mathcal{Q}_\sigma(\xi)\, \mathrm{d}\xi$$

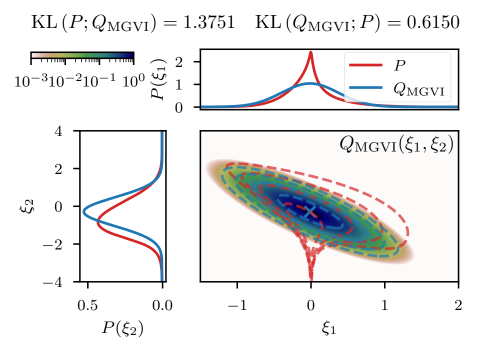
Posterior: $\mathcal{P}(\xi|d)$; Approximation: $\mathcal{Q}_\sigma(\xi)$; Variational parameters: σ.

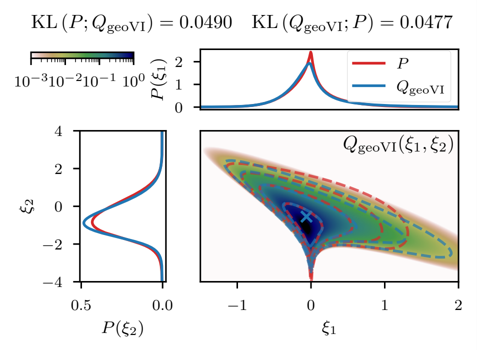
Variational Inference (VI)

$$\mathrm{KL}\,(P; Q_{\mathrm{MGVI}}) = 1.3751 \quad \mathrm{KL}\,(Q_{\mathrm{MGVI}}; P) = 0.6150$$



Variational Inference (VI)





Variational Inference (VI)

Kullback-Leibler divergence

$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = -\int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right)\ \mathcal{Q}_\sigma(\xi)\ \mathrm{d}\xi$$

Posterior: $\mathcal{P}(\xi|d)$; Approximation: $\mathcal{Q}_\sigma(\xi)$; Variational parameters: σ.

Variational Inference (VI)

Kullback-Leibler divergence

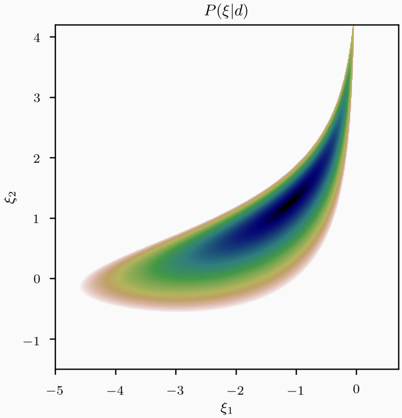
$$\mathrm{KL}\left[\mathcal{Q}_\sigma || \mathcal{P}\right] = -\int \log\left(\frac{\mathcal{P}(\xi|d)}{\mathcal{Q}_\sigma(\xi)}\right) \mathcal{Q}_\sigma(\xi)\, \mathrm{d}\xi$$

Posterior: $\mathcal{P}(\xi|d)$; Approximation: $\mathcal{Q}_\sigma(\xi)$; Variational parameters: σ.

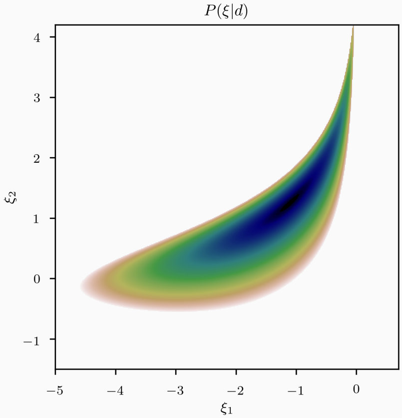
Approximate distribution $\mathcal{Q}$: $\mathcal{Q}(y) = \mathcal{N}(y; 0, \mathbb{1})$

Coordinate system $y = g_\sigma(\xi)$ such that the *posterior* is close to Normal.

Geometric Variational Inference (geoVI) [FLE21]



Information Hamiltonian $\mathcal{H}(\xi|d)$: $\quad -\log\left(\mathcal{P}(\xi|d)\right)$

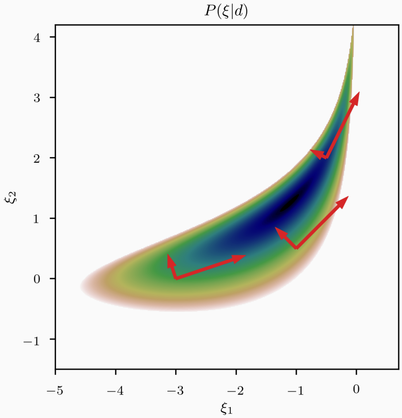


Information Hamiltonian $\mathcal{H}(\xi|d)$: $-\log\left(\mathcal{P}(\xi|d)\right)$

Posterior metric $\mathcal{M}(\xi)$: $\mathcal{M}_{\mathrm{lh}}(\xi) + \mathbb{1}$

Fisher information metric $\mathcal{M}_{\mathrm{lh}}(\xi)$: $\left\langle \frac{\partial^2 \mathcal{H}(d|\xi)}{\partial\xi\partial\xi'} \right\rangle_{\mathcal{P}(d|\xi)}$

Geometric Variational Inference (geoVI) [FLE21]

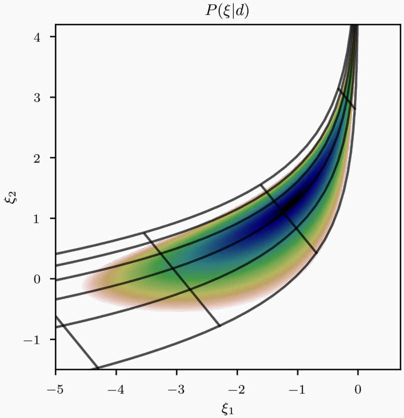


Information Hamiltonian $\mathcal{H}(\xi|d)$: $-\log\left(\mathcal{P}(\xi|d)\right)$

Posterior metric $\mathcal{M}(\xi)$: $\mathcal{M}_{\mathrm{lh}}(\xi) + \mathbb{1}$

Fisher information metric $\mathcal{M}_{\mathrm{lh}}(\xi)$: $\left\langle \frac{\partial^2 \mathcal{H}(d|\xi)}{\partial \xi \partial \xi'} \right\rangle_{\mathcal{P}(d|\xi)}$

Geometric Variational Inference (geoVI) [FLE21]

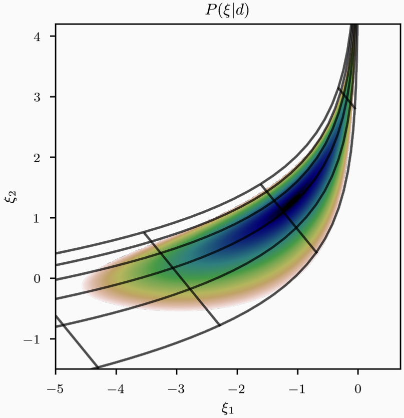


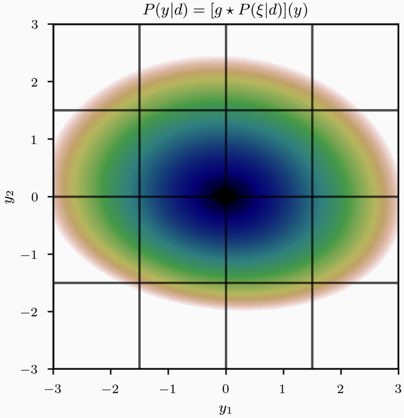
Information Hamiltonian $\mathcal{H}(\xi|d)$: $-\log\left(\mathcal{P}(\xi|d)\right)$

Posterior metric $\mathcal{M}(\xi)$: $\mathcal{M}_{\mathrm{lh}}(\xi) + \mathbb{1}$

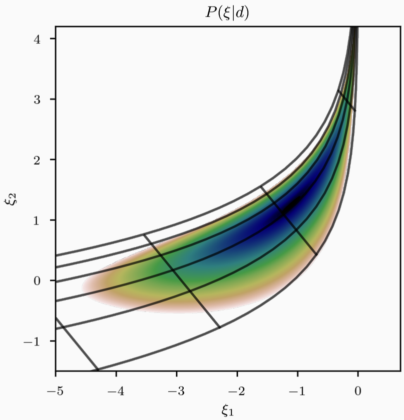
Fisher information metric $\mathcal{M}_{\mathrm{lh}}(\xi)$: $\left\langle \frac{\partial^2 \mathcal{H}(d|\xi)}{\partial\xi\partial\xi'} \right\rangle_{\mathcal{P}(d|\xi)}$

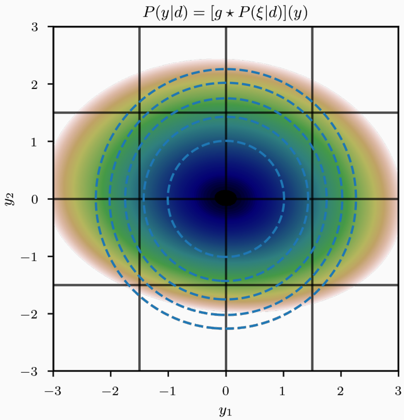
Geometric Variational Inference (geoVI) [FLE21]



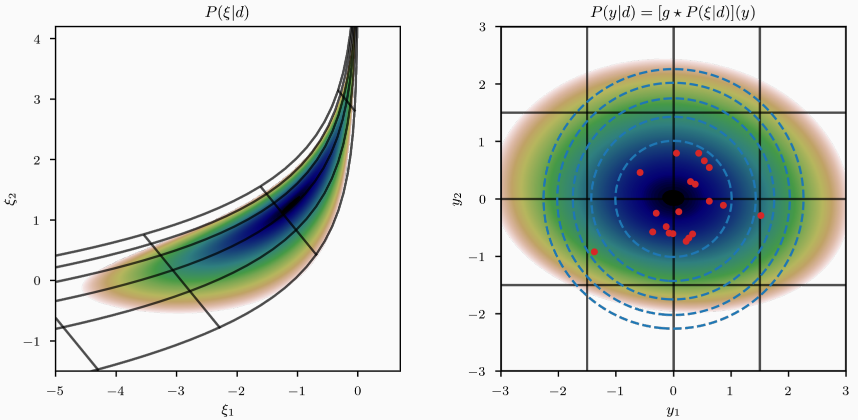


Geometric Variational Inference (geoVI) [FLE21]

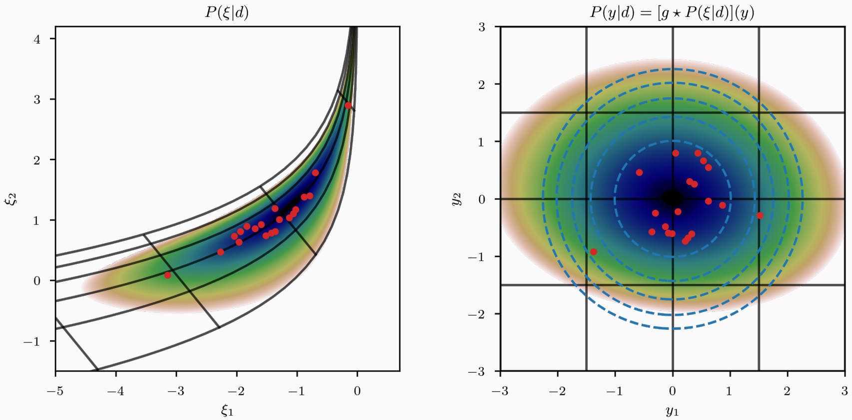




Geometric Variational Inference (geoVI) [FLE21]



Geometric Variational Inference (geoVI) [FLE21]



Geometric Variational Inference (geoVI) [FLE21]

Local Euclidean isometry around $\bar{\xi}$

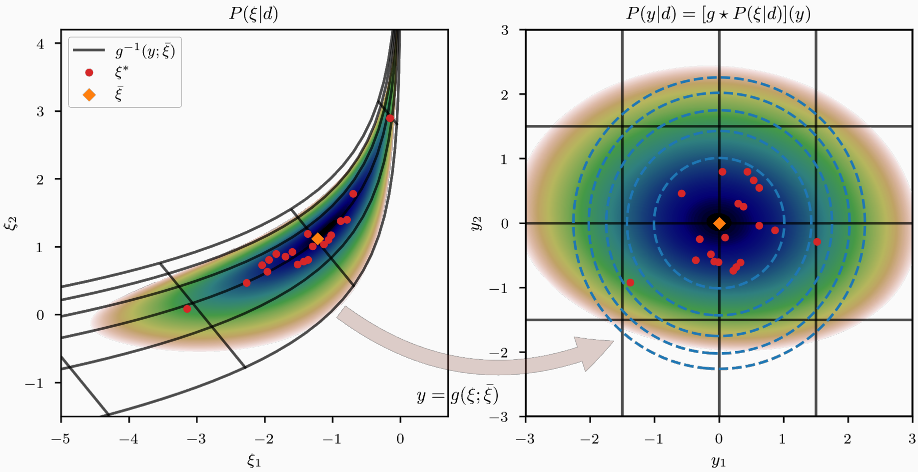
$$y = g(\xi; \bar{\xi}) = \mathcal{M}(\bar{\xi})^{-1/2} \left[\xi - \bar{\xi} + \left(\frac{\partial x}{\partial \xi} \right)^T \Bigg|_{\xi = \bar{\xi}} \left(x(\xi) - x(\bar{\xi}) \right) \right]$$

Likelihood transformation: $x(\xi) = x(s(\xi))$, expansion point: $\bar{\xi}$.

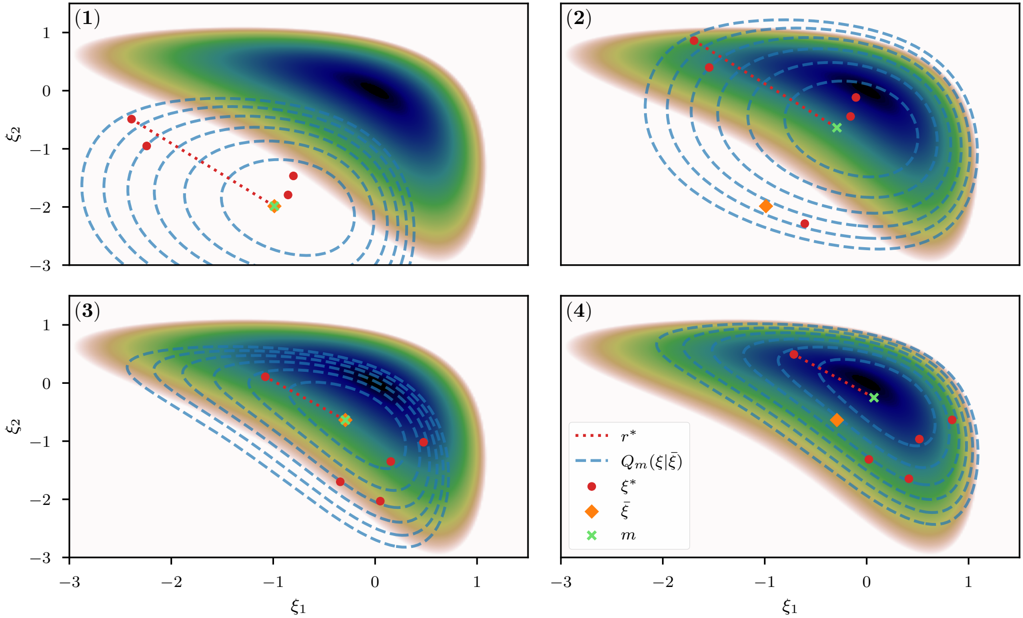
Variational approximation with transformed distribution $\mathcal{Q}$

$$\mathcal{Q}_{\bar{\xi}}(\xi) = \mathcal{N}(y|0, \mathbb{1})|_{y=g(\xi;\bar{\xi})} \left\| \frac{\partial g(\xi; \bar{\xi})}{\partial \xi} \right\|$$

Geometric Variational Inference (geoVI) [FLE21]



Geometric Variational Inference (geoVI) [FLE21]



References

Philipp Frank, Reimar Leike, and Torsten A. Enßlin.
Geometric variational inference.
Entropy, 23(7), 2021.