

# M87\* IN SPACE, TIME AND FREQUENCY

## DYNAMIC VLBI IMAGING WITH INFORMATION FIELD THEORY

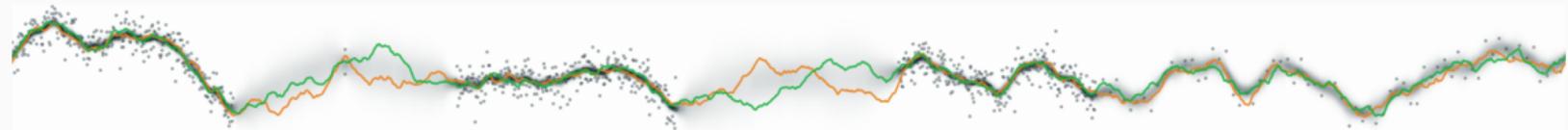
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Black Hole Initiative - Harvard University, November 7, 2022

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(2) Ludwig-Maximilians Universität LMU, Munich, Germany





## Product Rule of Probabilities

aka Bayes' theorem

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s) \mathcal{P}(s)}{\mathcal{P}(d)}$$

$\mathcal{P}(A|B)$ : conditional probability,  
s: parameters, d: data.

## (Some) assumptions

- The brightness is strictly positive.
- The source features correlation in spatial, temporal and frequency direction.  
⇒ Encoded in  $\mathcal{P}(s)$ .

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In our case

- Correlation structure → full 4d-movie: sky brightness has shape (2, 28, 256, 256).
- The posterior  $\mathcal{P}(s|d)$  is a ridiculously high-dimensional function:

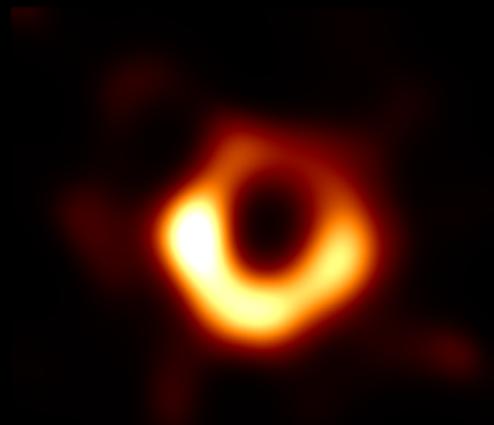
$$\begin{cases} \mathbb{R}^{7,500,000} & \rightarrow \mathbb{R}^{\geq 0} \\ s & \mapsto \mathcal{P}(s|d) \end{cases}$$

- This function encodes our knowledge on M87\* including uncertainties.

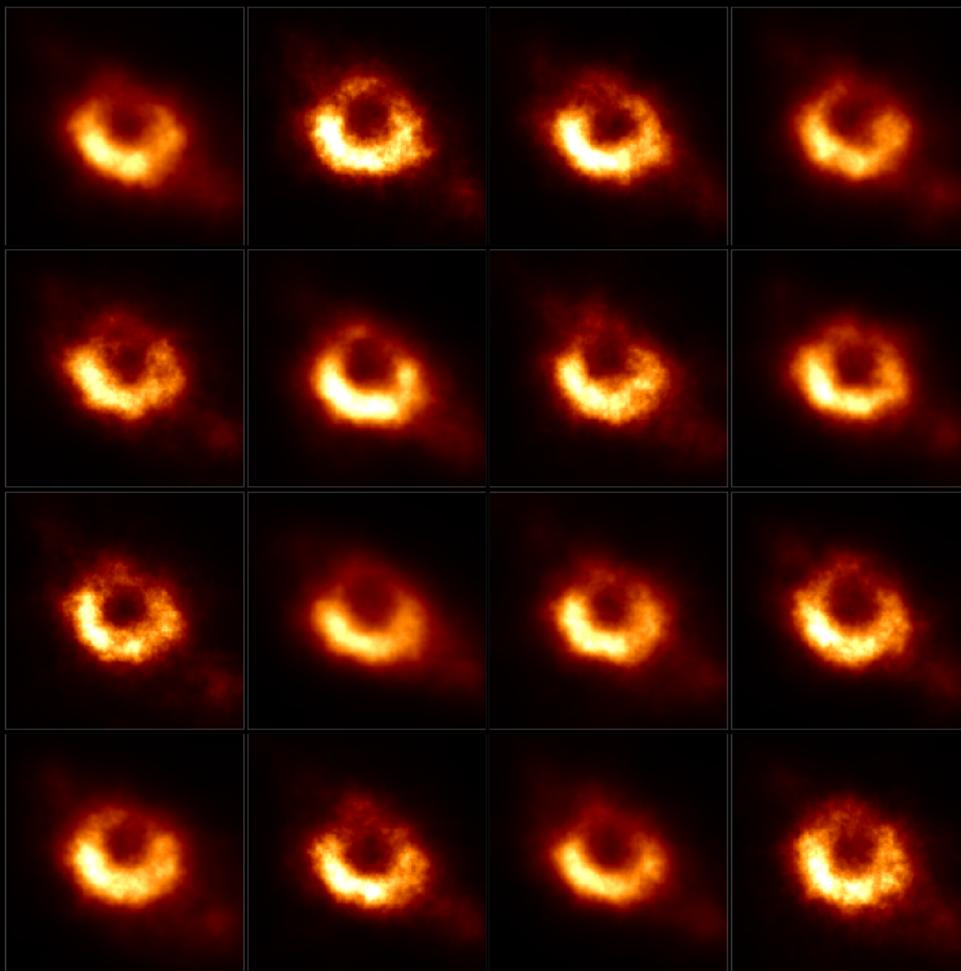
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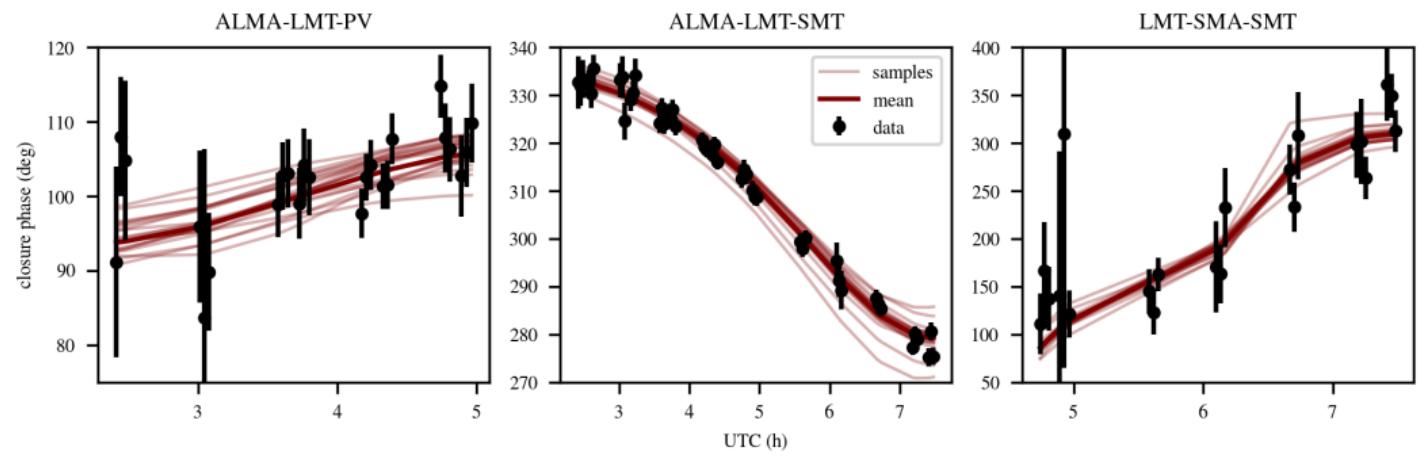






vlbi-resolve, [AFH<sup>+</sup>22], 16 posterior samples.

# DATA CONSISTENCY

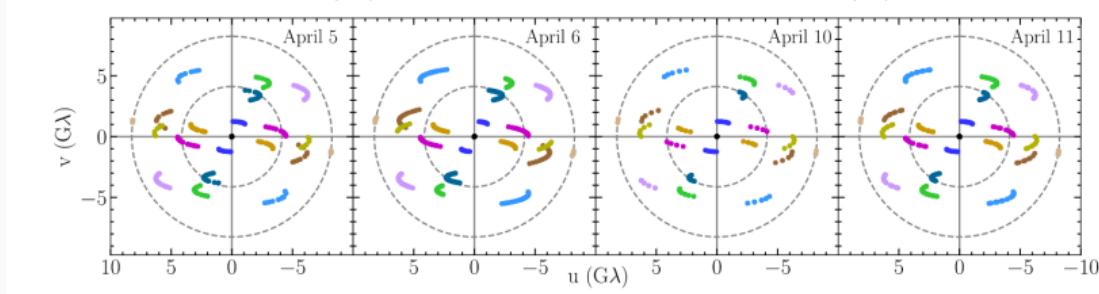


**Figure 1:** Three closure phases for triples of antennas as a function of time.

## SETUP

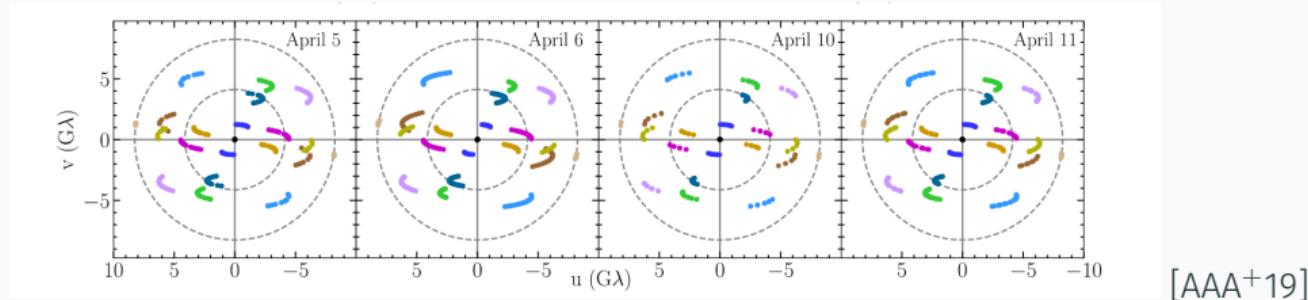
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# THE EVENT HORIZON TELESCOPE (EHT)



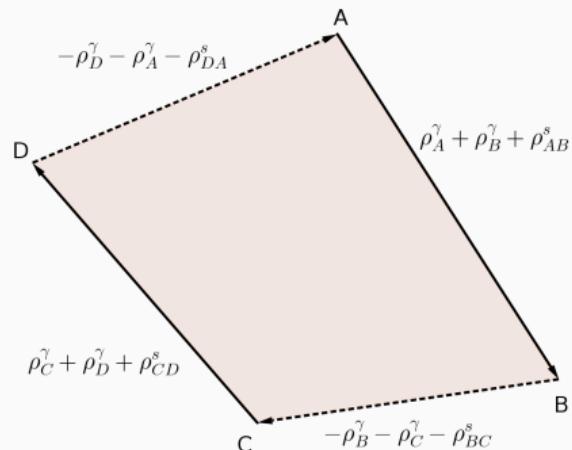
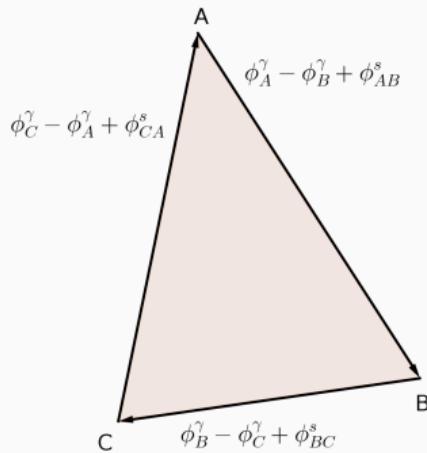
[AAA<sup>+</sup>19]

# THE EVENT HORIZON TELESCOPE (EHT)



# LIKELIHOOD

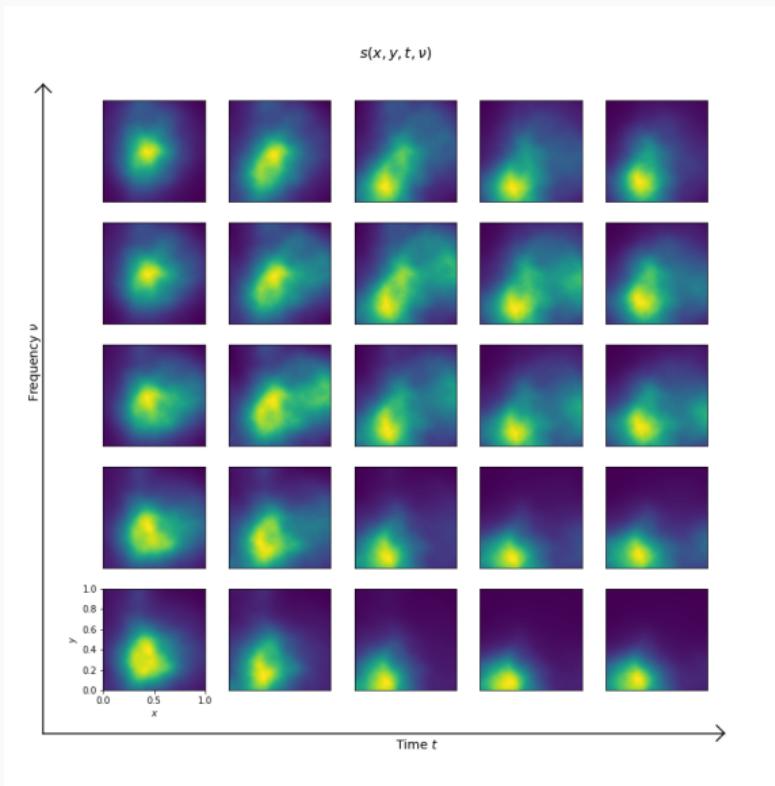
⇒ Imaging using only closure quantities (phases  $\phi^d$  and logarithmic amplitudes  $\rho^d$ )



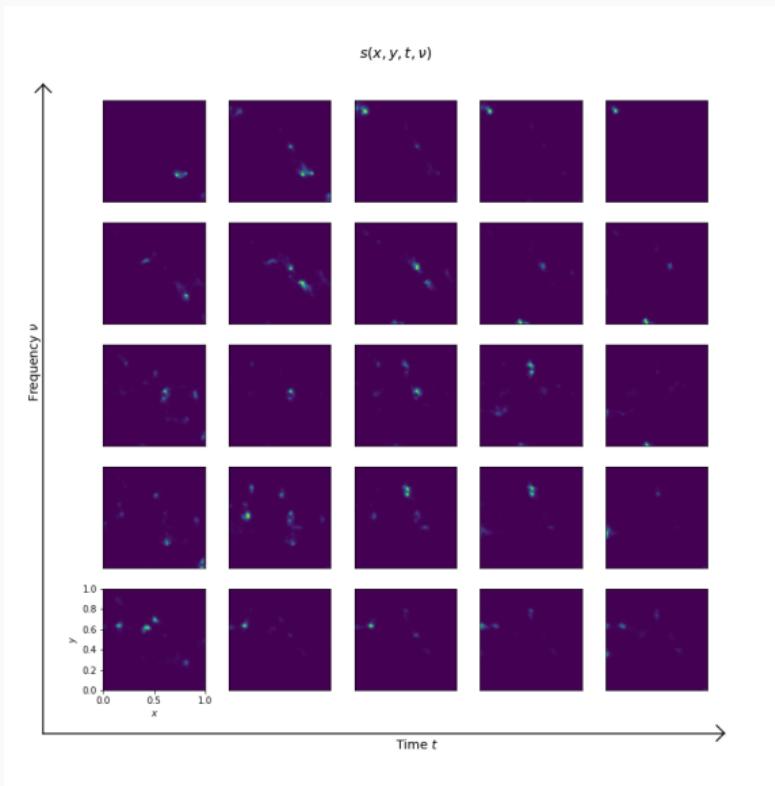
$$\phi_{\text{clos}}^d = \textcolor{brown}{M}\phi^d$$

$$\rho_{\text{clos}}^d = \textcolor{brown}{L}\rho^d$$

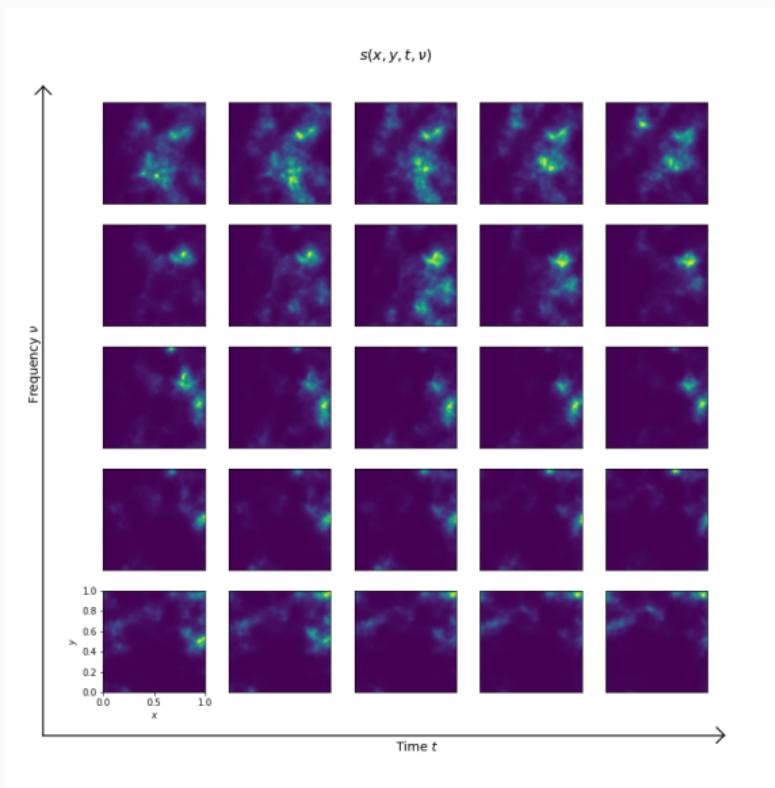
# PRIOR - CORRELATIONS

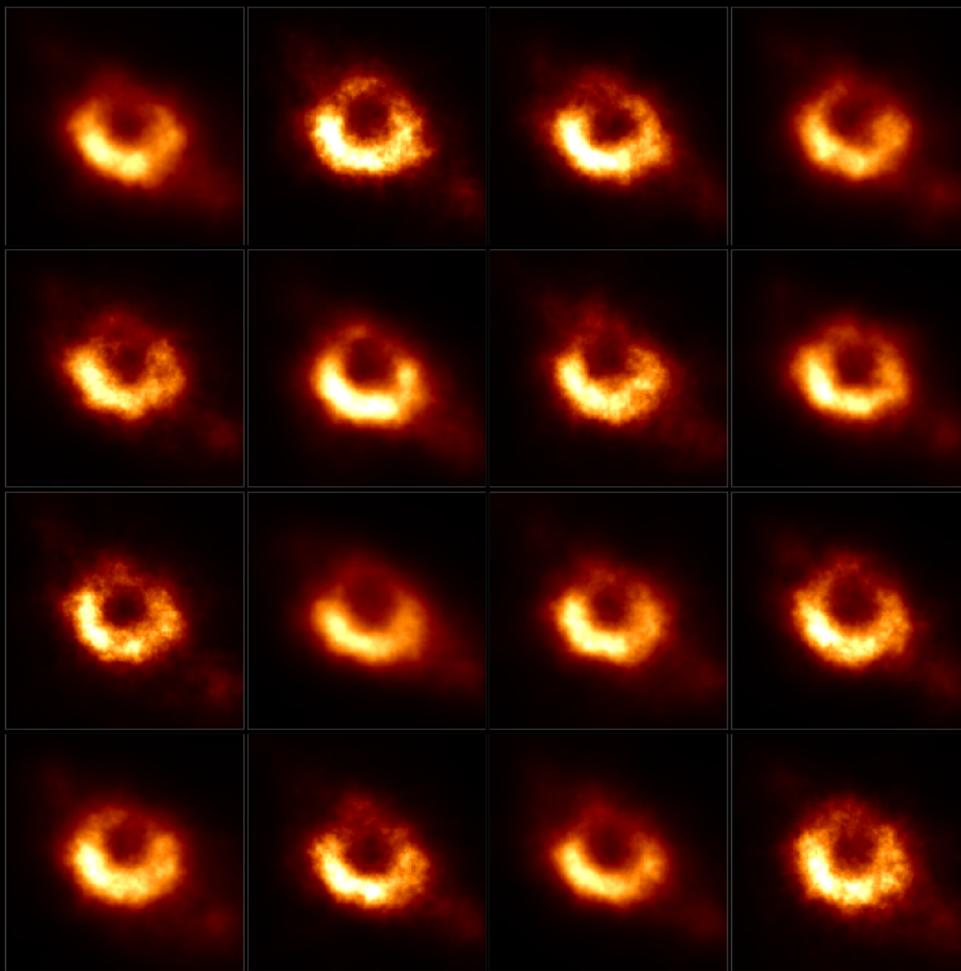


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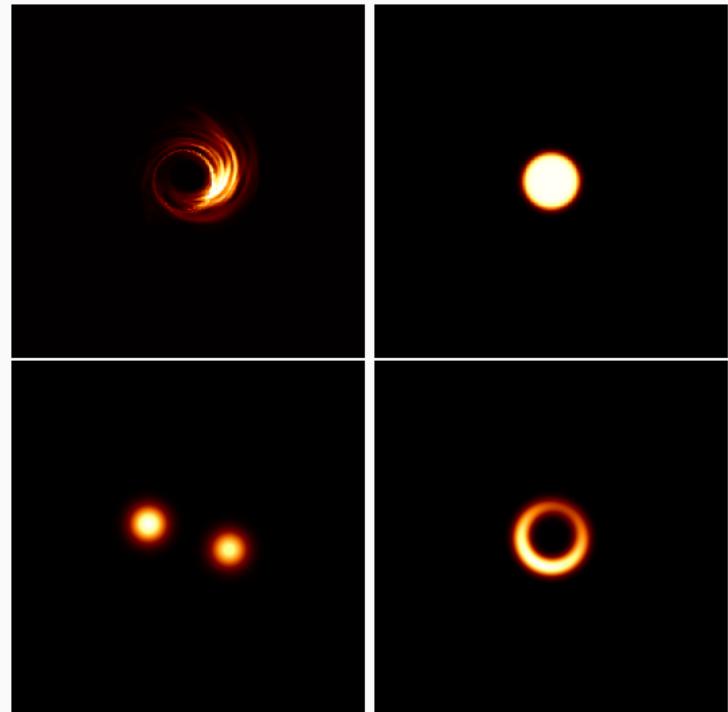
vlbi-resolve, [AFH<sup>+</sup>22], 16 posterior samples.

## VALIDATION

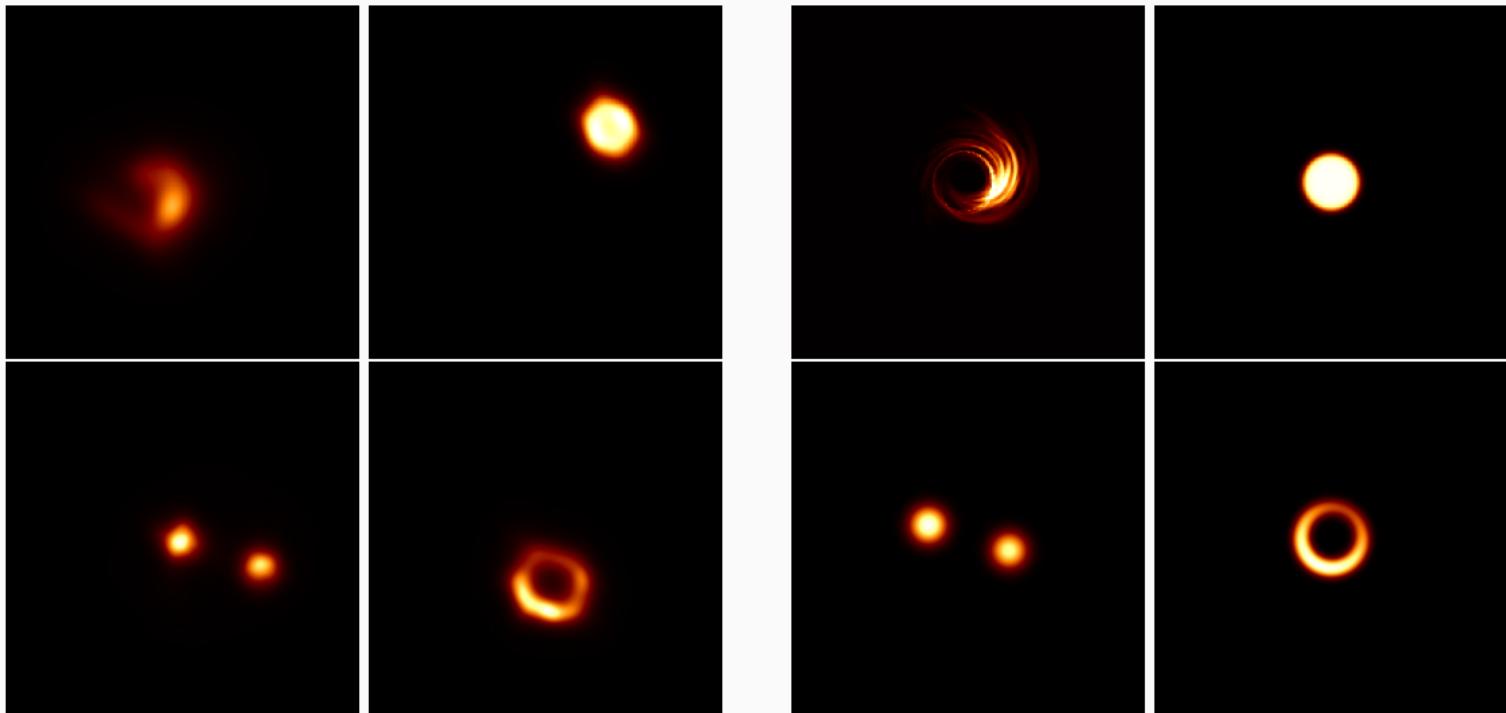
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## VALIDATION

- Synthetic source
- Generate data according to EHT observation
- Reconstruct
- Compare to truth

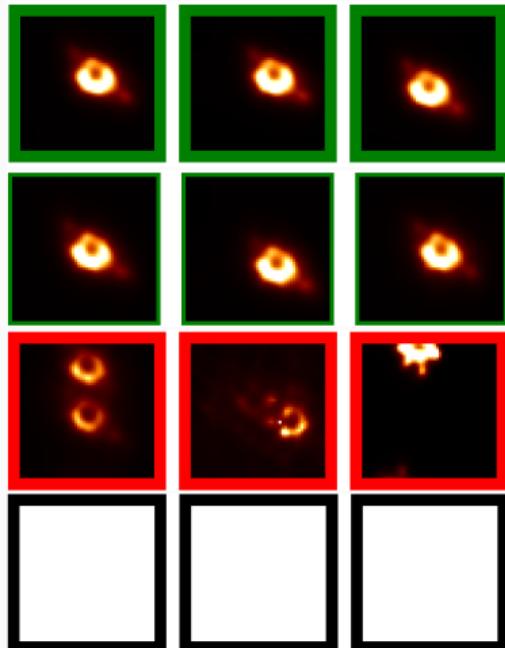


## VALIDATION

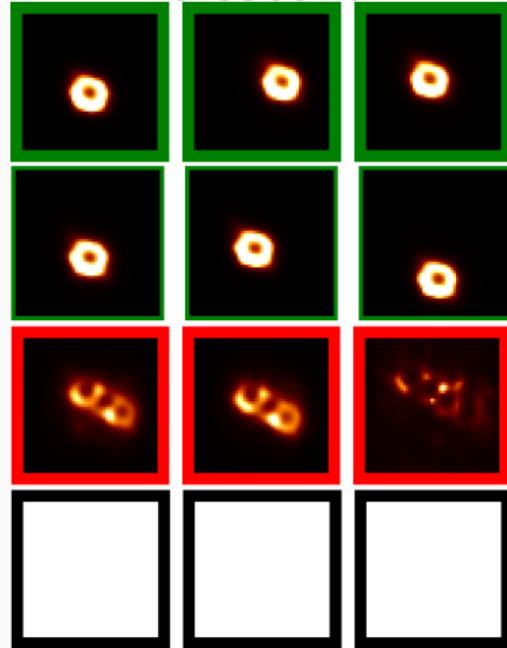


## HYPERPARAMETER VALIDATION - THE GOOD, THE BAD AND THE UGLY

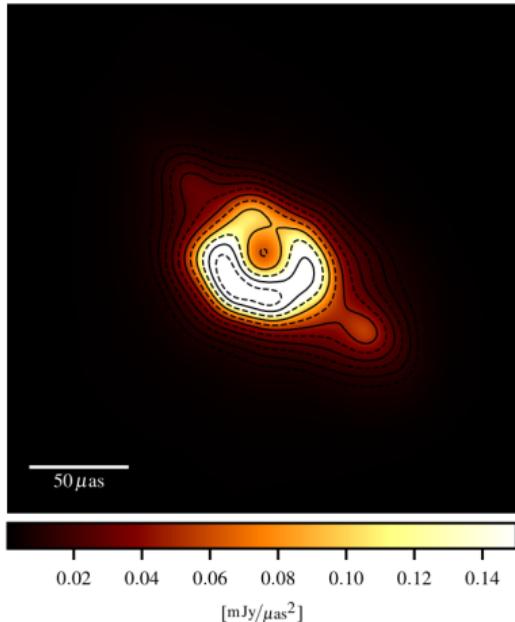
M87\*



Crescent



# CONCLUSION



**Figure 2:** M87\* on day 0 imaged with **vlbi-resolve** [AFH<sup>+</sup>22]. Saturated color bar.

## Key aspects

- **Four-dimensional** (time, frequency, space) reconstruction of M87\*
- **Correlation kernel** is non-parametrically learned from the data
- **Bayesian** treatment despite huge problem size ( $10^7$  dofs)
- **Intrinsic** uncertainty quantification

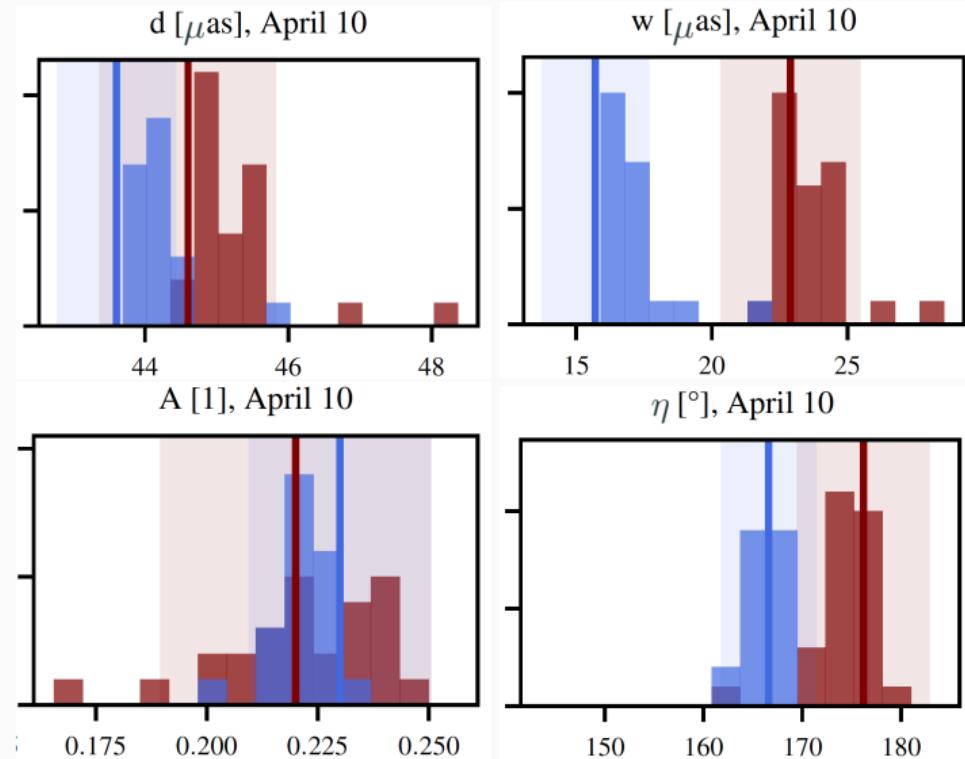
## REFERENCES

-  Kazunori Akiyama, Antxon Alberdi, Walter Alef, Keiichi Asada, Rebecca Azulay, Anne-Kathrin Baczko, David Ball, Mislav Baloković, John Barrett, Dan Bintley, et al.  
**First M87 Event Horizon Telescope Results. IV. Imaging the Central Supermassive Black Hole.**  
*The Astrophysical Journal Letters*, 875(1):L4, April 2019.
-  Philipp Arras, Philipp Frank, Philipp Haim, Jakob Knollmüller, Reimar Leike, Martin Reinecke, and Torsten Enßlin.  
**Variable structures in m87\* from space, time and frequency resolved interferometry.**  
*Nature Astronomy*, 6(2):259–269, 2022.

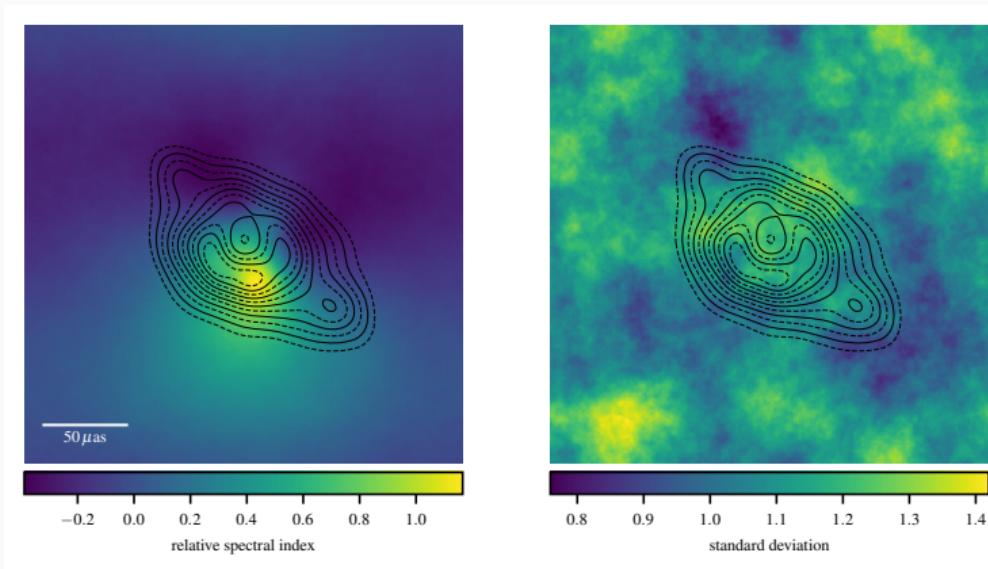
## RING FITTING (SEE [AAA<sup>+</sup>19, TABLE 7])

	$d$ ( $\mu$ as)	$w$ ( $\mu$ as)	$\eta$ ( $^{\circ}$ )	A	$f_c$
EHT-IMAGING [AAA <sup>+</sup> 19]					
April 5	$39.3 \pm 1.6$	$16.2 \pm 2.0$	$148.3 \pm 4.8$	$0.25 \pm 0.02$	0.08
April 6	$39.6 \pm 1.8$	$16.2 \pm 1.7$	$151.1 \pm 8.6$	$0.25 \pm 0.02$	0.06
April 10	$40.7 \pm 1.6$	$15.7 \pm 2.0$	$171.2 \pm 6.9$	$0.23 \pm 0.03$	0.04
April 11	$41.0 \pm 1.4$	$15.5 \pm 1.8$	$168.0 \pm 6.9$	$0.20 \pm 0.02$	0.04
OUR METHOD					
UNCERTAINTY AS PER [AAA <sup>+</sup> 19, TABLE 7])					
April 5	$44.4 \pm 3.4$	$23.2 \pm 5.2$	$164.9 \pm 9.5$	$0.26 \pm 0.04$	0.365
April 6	$44.4 \pm 2.9$	$23.3 \pm 5.4$	$161.7 \pm 5.6$	$0.24 \pm 0.04$	0.374
April 10	$44.8 \pm 2.8$	$23.0 \pm 5.0$	$176.7 \pm 9.8$	$0.22 \pm 0.03$	0.374
April 11	$44.6 \pm 2.8$	$22.8 \pm 4.8$	$180.1 \pm 10.4$	$0.22 \pm 0.03$	0.372
SAMPLE UNCERTAINTY					
April 5	$44.1 \pm 1.2$	$23.1 \pm 2.4$	$163.9 \pm 5.0$	$0.25 \pm 0.03$	$0.377 \pm 0.081$
April 6	$44.0 \pm 1.2$	$22.9 \pm 2.4$	$161.9 \pm 6.0$	$0.24 \pm 0.03$	$0.385 \pm 0.085$
April 10	$44.6 \pm 1.2$	$22.9 \pm 2.5$	$176.2 \pm 6.5$	$0.22 \pm 0.03$	$0.383 \pm 0.089$
April 11	$44.6 \pm 1.2$	$23.0 \pm 2.6$	$179.8 \pm 6.2$	$0.22 \pm 0.03$	$0.383 \pm 0.090$

# RING PARAMETERS



## SPECTRAL DEPENDENCY



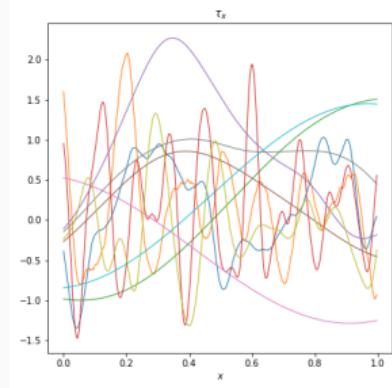
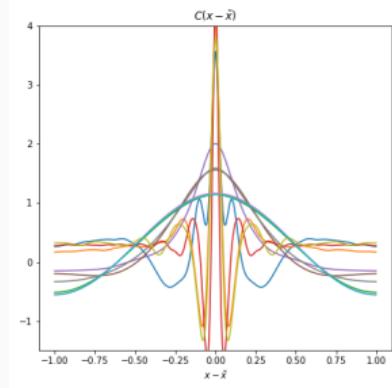
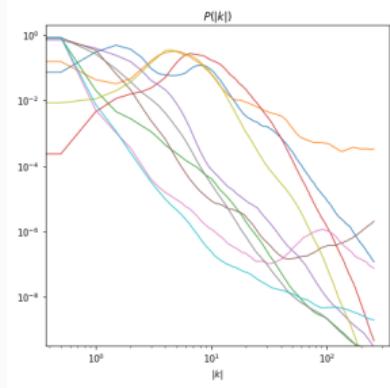
**Figure 3:** The relative spectral index and the pixel-wise uncertainty, as calculated from the 227–229 GHz channels.

## REDUCED $\chi^2$

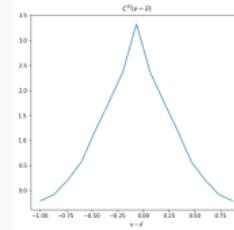
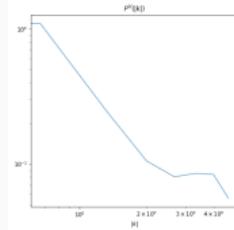
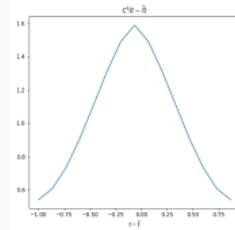
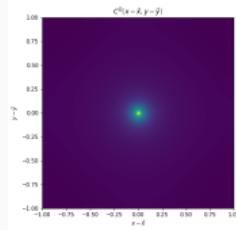
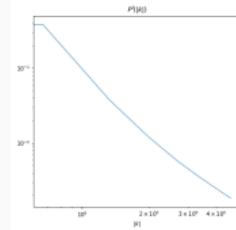
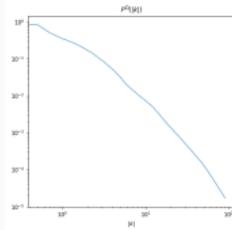
	April 5	April 6	April 10	April 11
Simulation	1.2, 1.0	1.3, 1.2	1.4, 1.3	1.1, 1.1
Disk	1.6, 1.2	1.4, 1.3	1.5, 1.4	1.3, 1.2
Double Sources	1.2, 1.1	1.2, 1.1	1.3, 1.3	1.4, 1.1
Crescent	1.2, 1.0	1.3, 0.9	1.0, 0.9	1.4, 1.1
M87*	1.1, 0.9	1.1, 0.8	1.1, 0.9	1.1, 0.9

Table 1: The  $\chi^2$  of the reconstruction for closure (phase, amplitude).

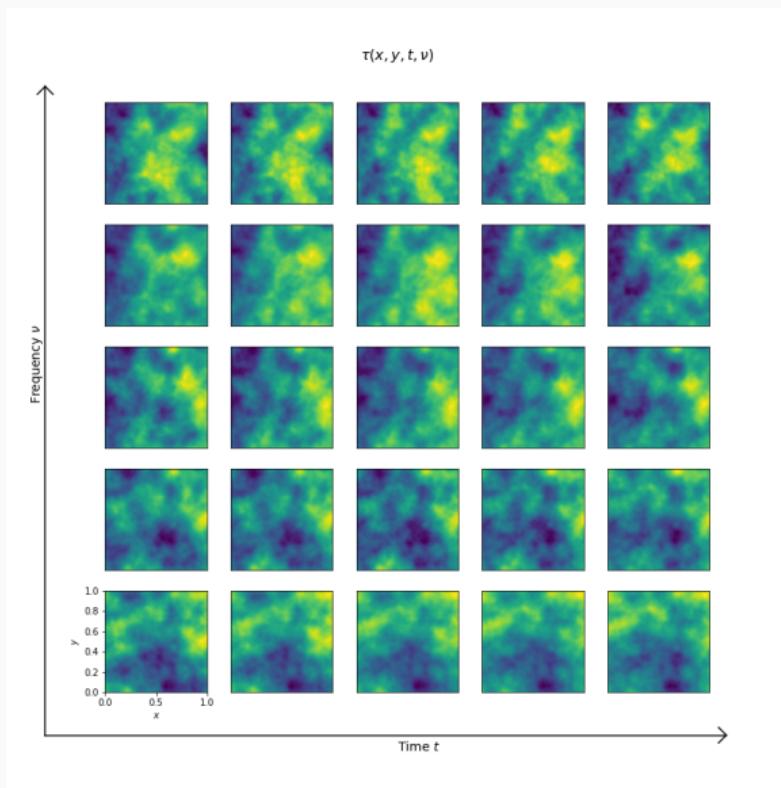
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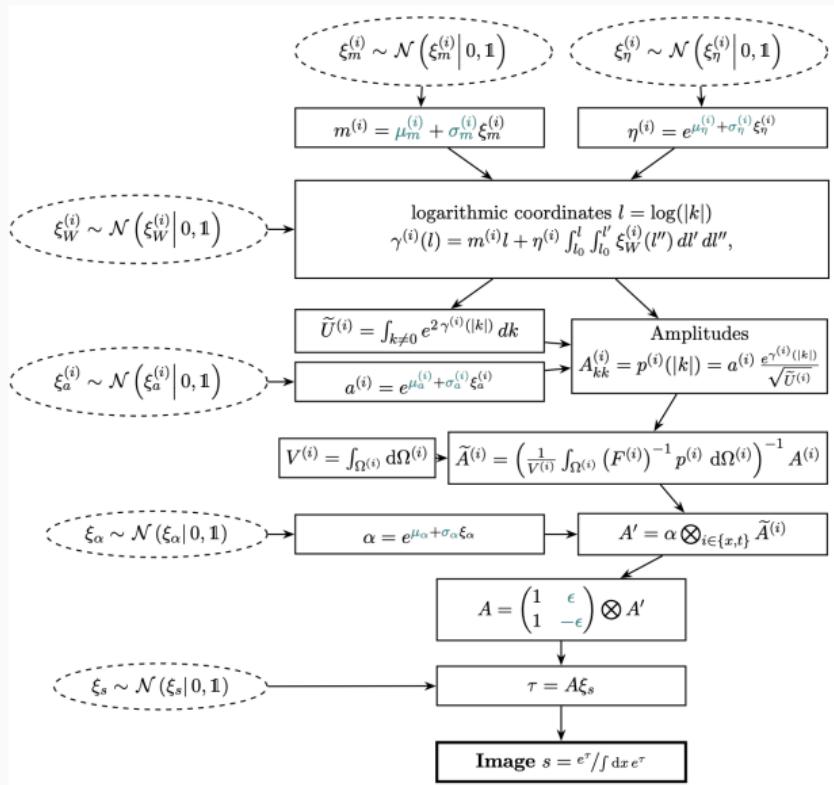
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# PRIOR



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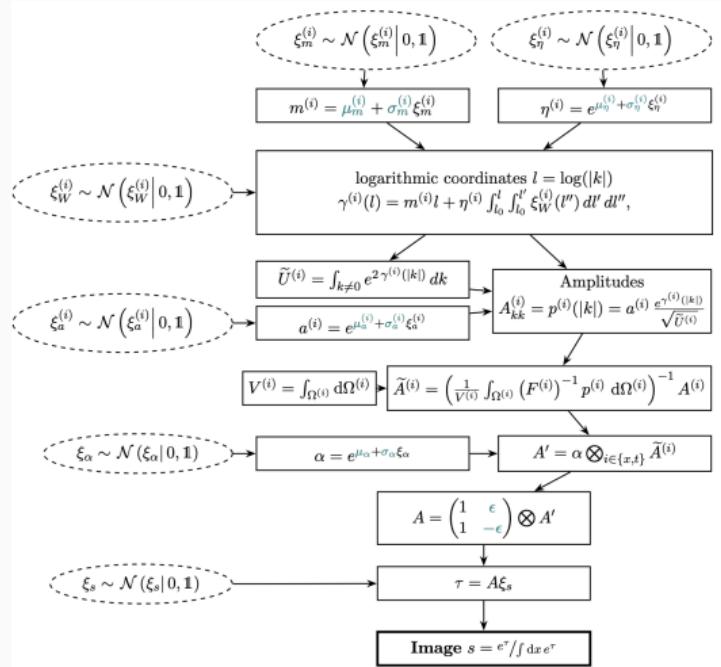
ehtimaging, [AAA<sup>+</sup>19], day 5.



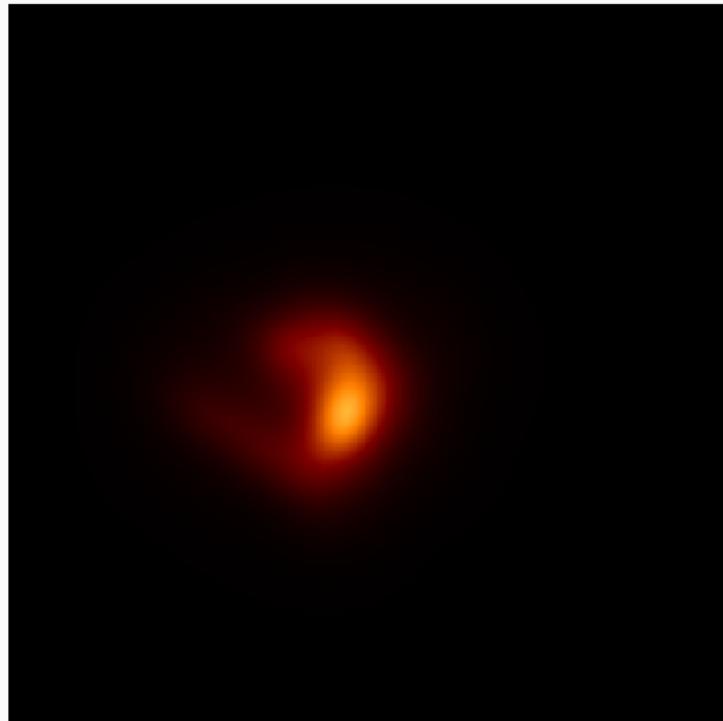
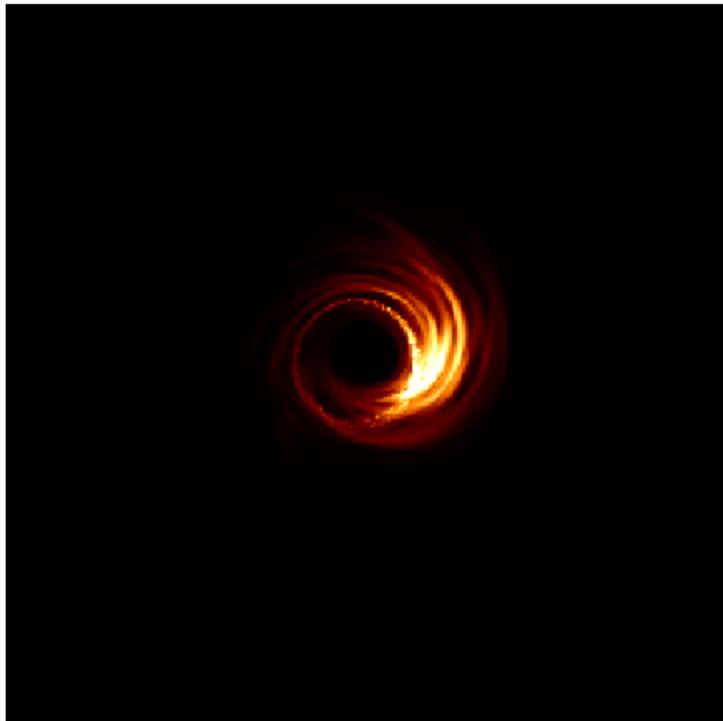
ehtimaging, [AAA<sup>+</sup>19], day 1.

# HYPERPARAMETER VALIDATION

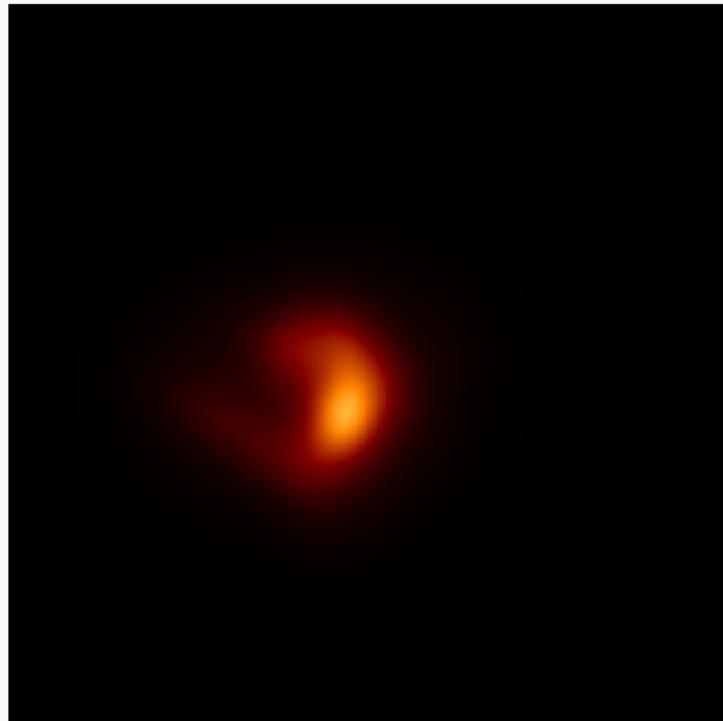
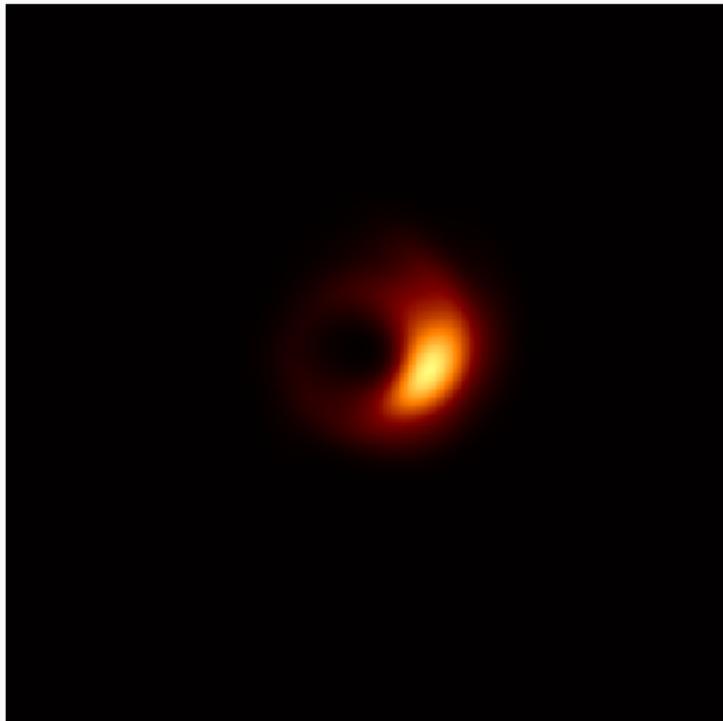
- In total 15 hyperparameters
- Specifying mean and variance
- Draw mean hyperparameters within a uniform  $3\sigma$  interval
- Perform 100 reconstructions



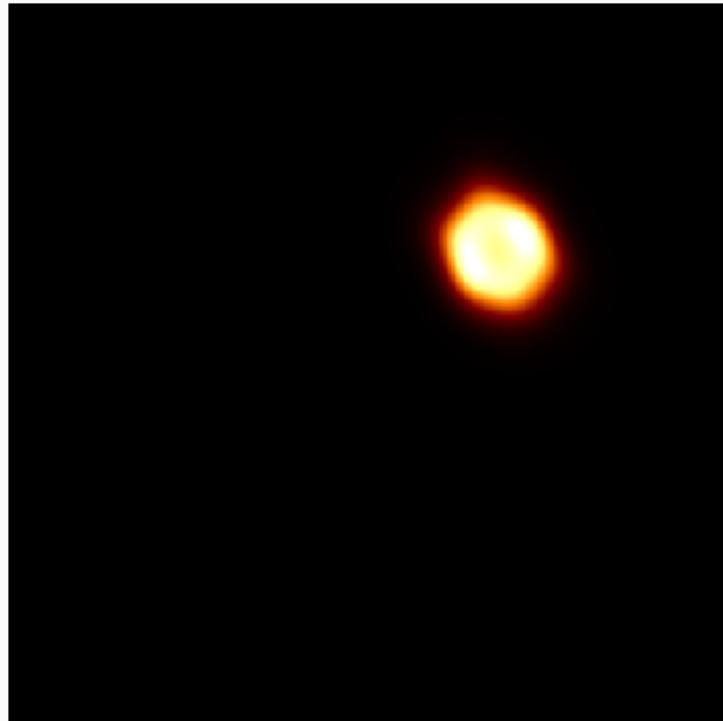
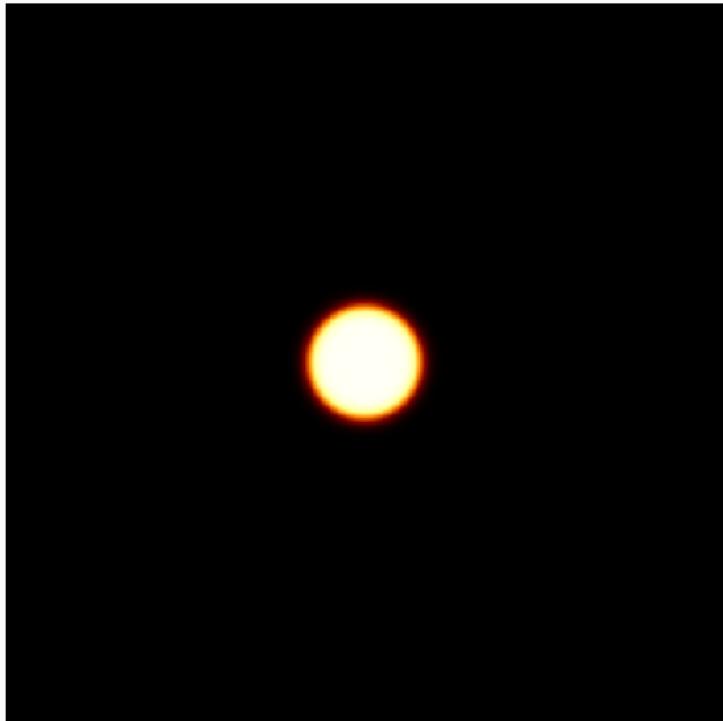
## STATIC SOURCE: SIMULATION



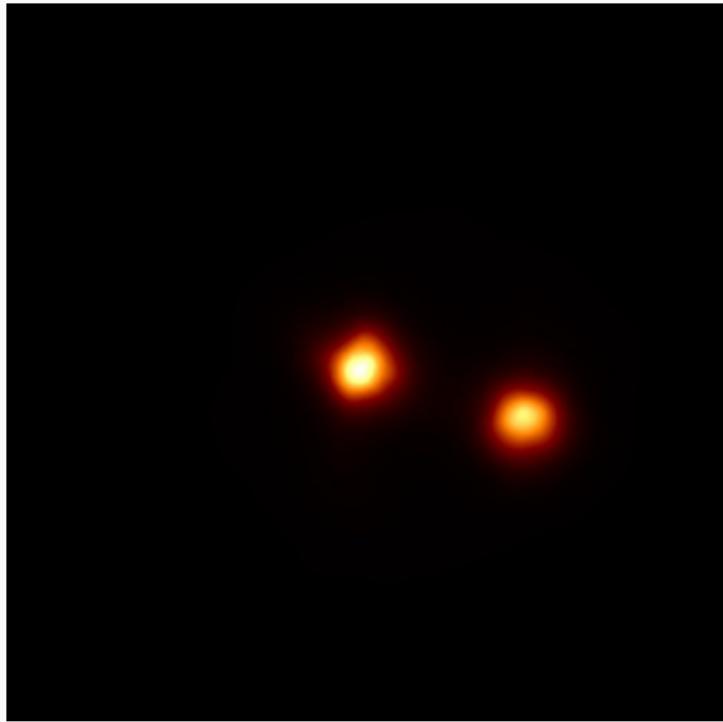
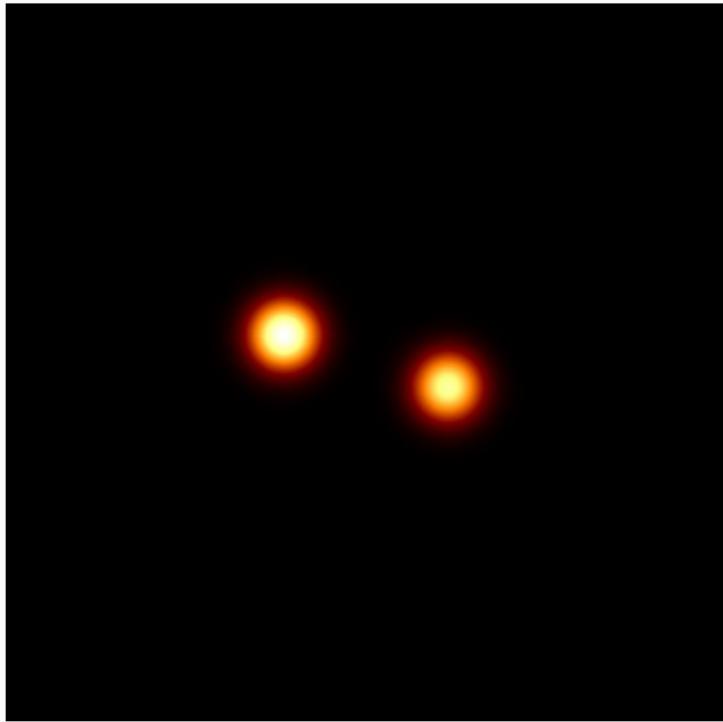
## STATIC SOURCE: SIMULATION



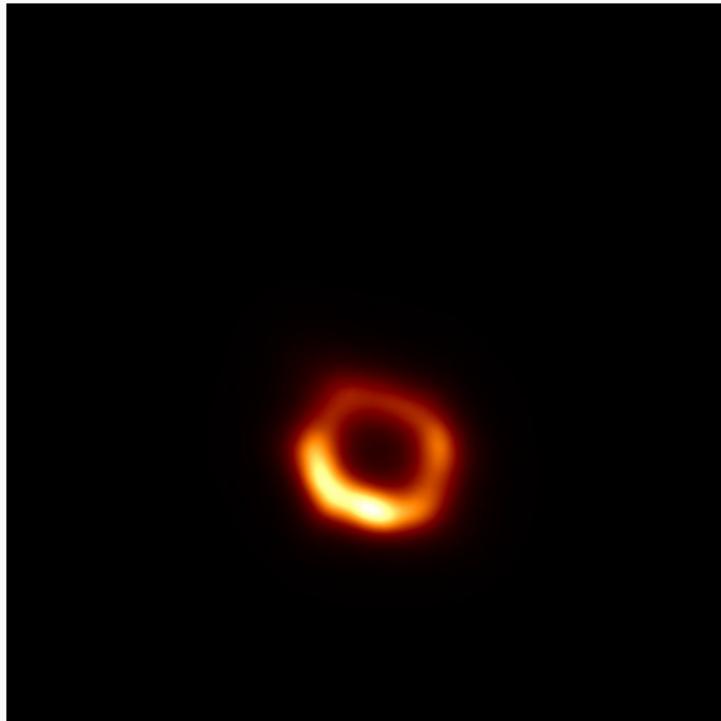
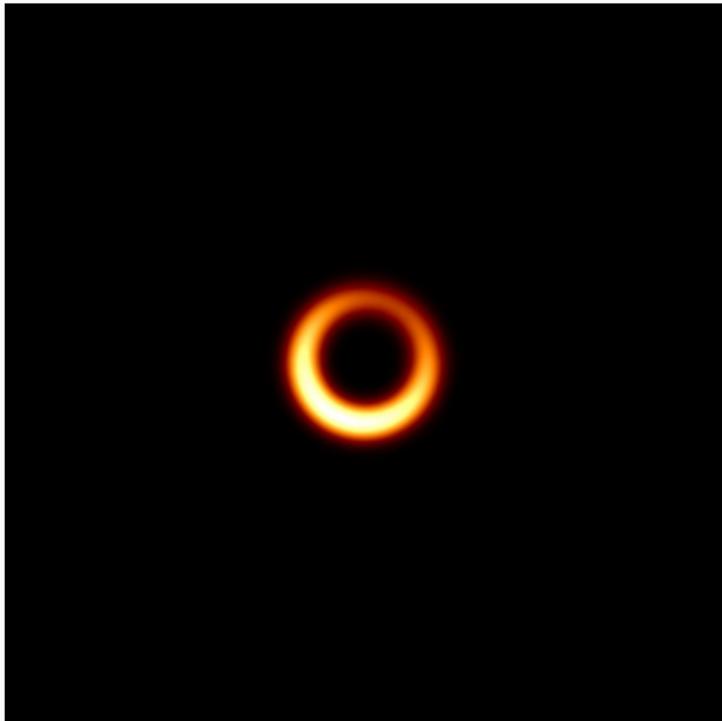
## STATIC SOURCE: DISK



## DYNAMIC SOURCE: GAUSSIAN SHAPES



# DYNAMIC SOURCE: CRESCENT



## VALIDATION

- Reconstruction works on various sources
- We recover dynamics
- Results are widely insensitive to hyperparameters
- Room for improvement in the inference heuristic

