Time variable and spectral resolved Bayesian Imaging and the variable shadow of M87*

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Bayesian Inference

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Bayes Theorem

$$P(s|d,M) = \frac{P(s,d|M)}{P(d|M)} = \frac{P(s,d|M)}{\int P(s,d|M)\ \mathrm{d}s}$$

Bayes Theorem $\Leftrightarrow$ Generative Models

$$P(s|d,M) = \frac{P(s,d|M)}{P(d|M)} = \frac{P(s,d|M)}{\int P(s,d|M)\,\mathrm{d}s}$$

Transformation rule for probabilities

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$$P(s,d|M) \quad \Leftrightarrow \quad P(\xi,d|M) = P(d|s(\xi),M)\,\mathcal{G}(\xi,\mathbb{1})$$

Modeling the sky brightness

$$s_{xt\nu} = e^{\tau_{xt\nu}} \quad \text{with} \quad x \in \Omega \subset \mathbb{R}^d,\ t \in I \subset \mathbb{R},\ \nu \in V \subset \mathbb{R}^+$$

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Exploit correlations

$$\left\langle \tau_{x,t,\nu}\ \tau_{x',t',\nu'} \right\rangle = C(x,t,\nu,x',t',\nu')$$

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$$\begin{aligned} \left\langle \tau_{x,t,\nu}\ \tau_{x',t',\nu'} \right\rangle &= C(x, t, \nu, x', t', \nu') \\ &= C\left(\left|x - x'\right|, \left|t - t'\right|, \left|\nu - \nu'\right|\right) \\ &= C^{\Omega}\left(\left|x - x'\right|\right)\ C^{I}\left(\left|t - t'\right|\right)\ C^{V}\left(\left|\nu - \nu'\right|\right) \end{aligned}$$

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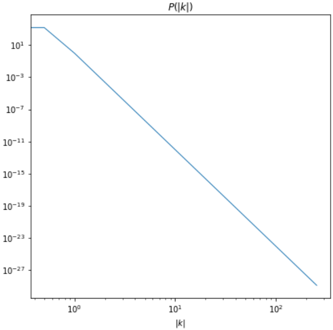
$$P(\tau|C) = \mathcal{G}(\tau, C) \quad \Leftrightarrow \quad \tau = \sqrt{C}\ \xi_\tau, \quad \xi_\tau \sim \mathcal{G}(\xi_\tau, \mathbb{1})$$

Modeling the sky brightness

$$P^{(i)}(|k|) \qquad\qquad i \in \{\Omega, I, V\}$$

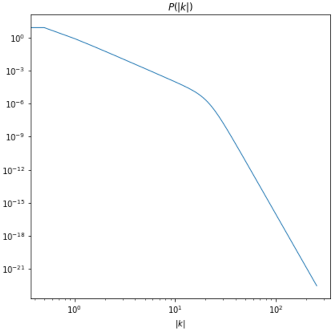
Modeling the sky brightness

$$P^{(i)}(|k|) \propto |k|^{-\alpha} \qquad i \in \{\Omega, I, V\}$$



Modeling the sky brightness

$P^{(i)}(|k|)$



Modeling the sky brightness

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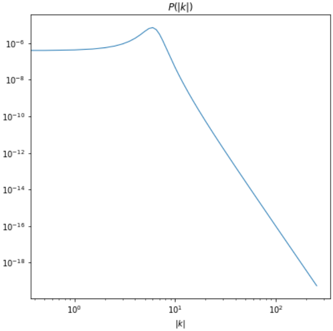

Modeling the sky brightness

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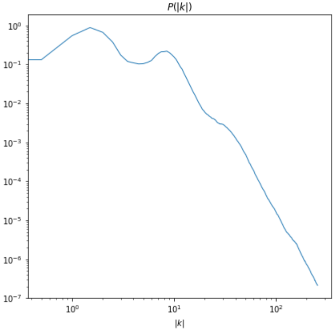

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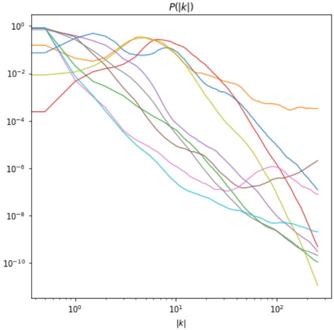

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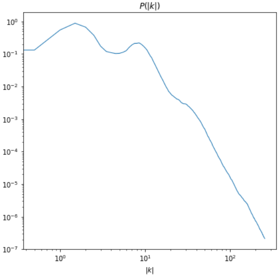

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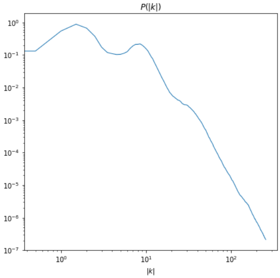

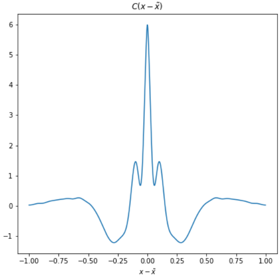

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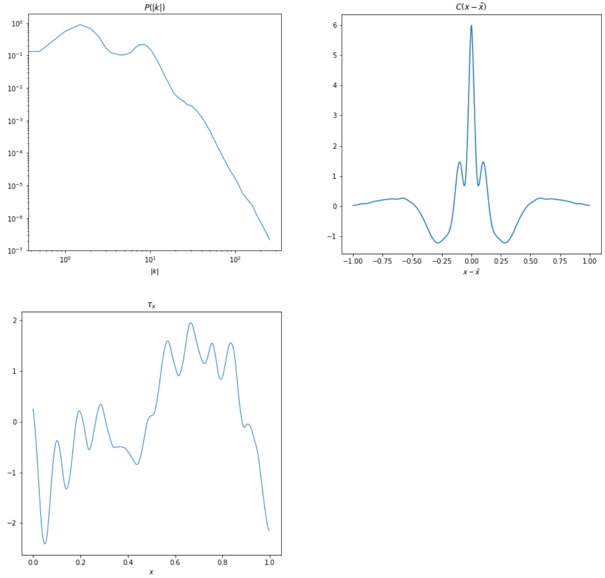

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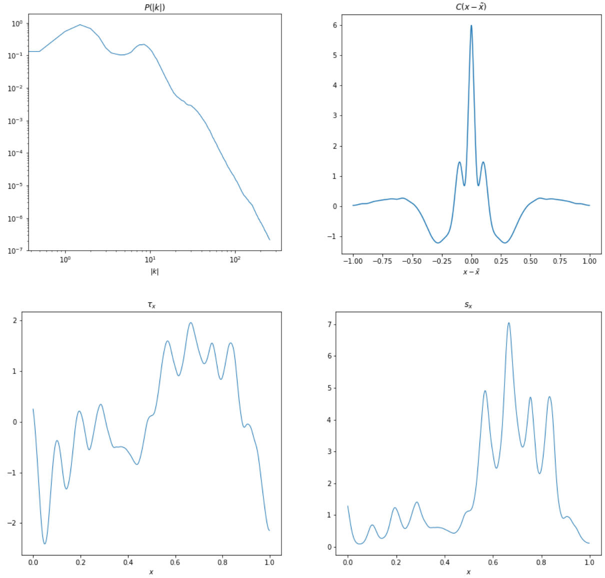

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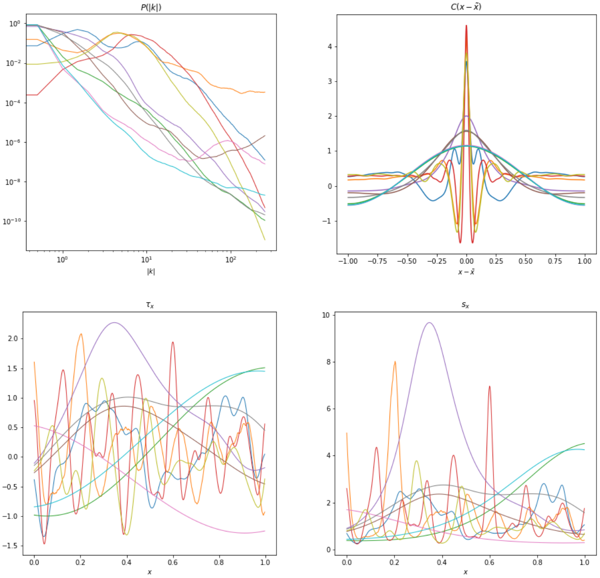

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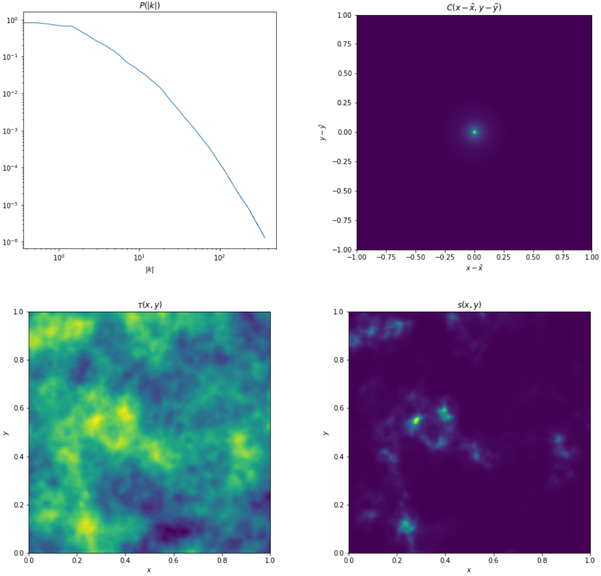

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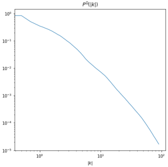

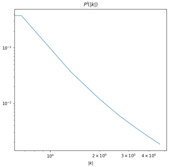

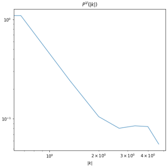

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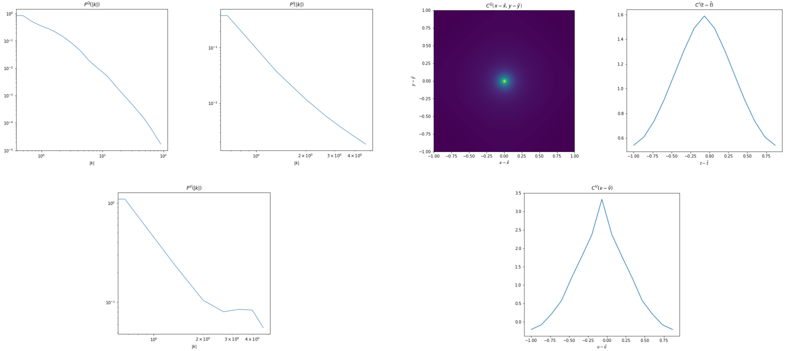

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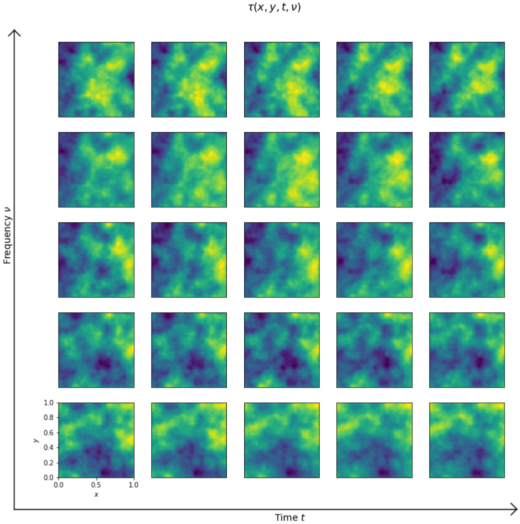

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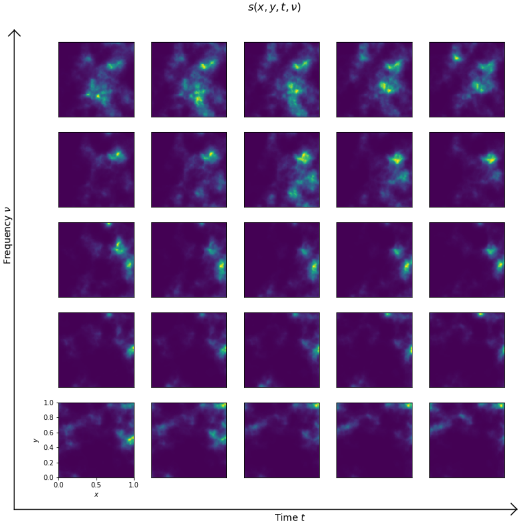
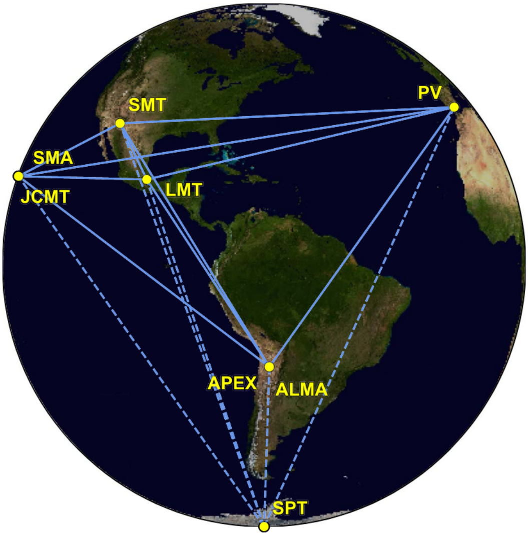


Time Variable Imaging of M87*

- Data provided by the Event Horizon Telescope (EHT)
- Extremely sparse measurements ($\sim$ 4100 Visibilities in total)
- Significant temporal variability on observational scales
- Averaged down to two frequency bands at 227 and 229 Ghz

The Event Horizon Telescope (EHT)



[1]

[1] Event Horizon Telescope Collaboration et al., "First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole".

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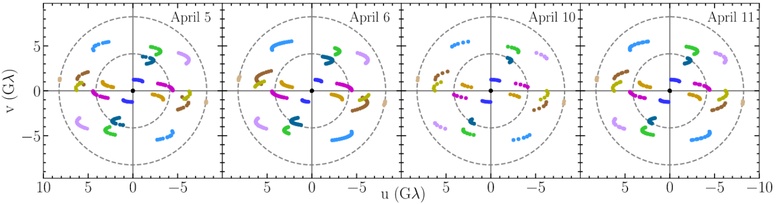

2

[2] Event Horizon Telescope Collaboration et al., "First M87 Event Horizon Telescope Results. IV. Imaging the Central Supermassive Black Hole".

Likelihood

$$(Rs)_{AB} = \int e^{-2\pi i(u_{AB}x + v_{AB}y)}\ s_{x,y}\ \mathrm{d}x = e^{\rho^s_{AB} + i\phi^s_{AB}}$$

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$$d_{AB} = \gamma_A \gamma_B^* \left(Rs\right)_{AB} + n_{AB} \qquad \text{with} \quad n_{AB} \sim \mathcal{G}\left(n_{AB}, \sigma^2\right)$$

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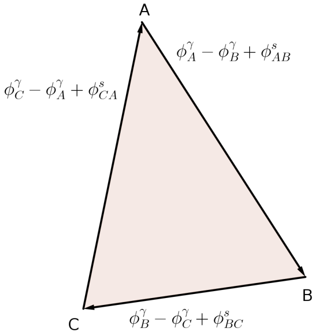
$$\phi^d_{AB} \approx \phi^\gamma_A - \phi^\gamma_B + \phi^s_{AB} + n^\phi_{AB} \quad \text{with} \quad n^\phi_{AB} \sim \mathcal{G}\left(n^\phi_{AB}, \frac{\sigma^2}{|d|^2}\right)$$

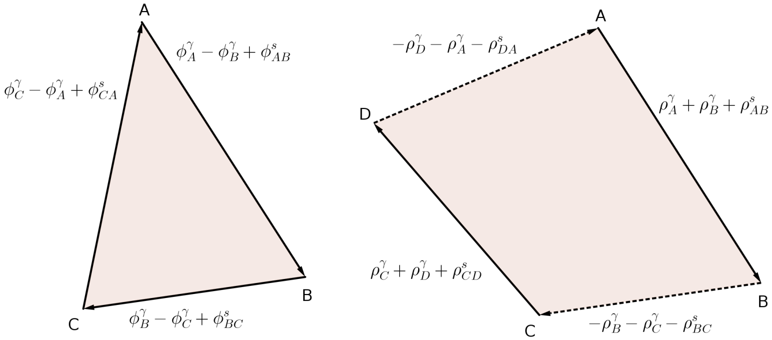
$$\rho^d_{AB} \approx \rho^\gamma_A + \rho^\gamma_B + \rho^s_{AB} + n^\rho_{AB} \quad \text{with} \quad n^\rho_{AB} \sim \mathcal{G}\left(n^\rho_{AB}, \frac{\sigma^2}{|d|^2}\right)$$

Likelihood



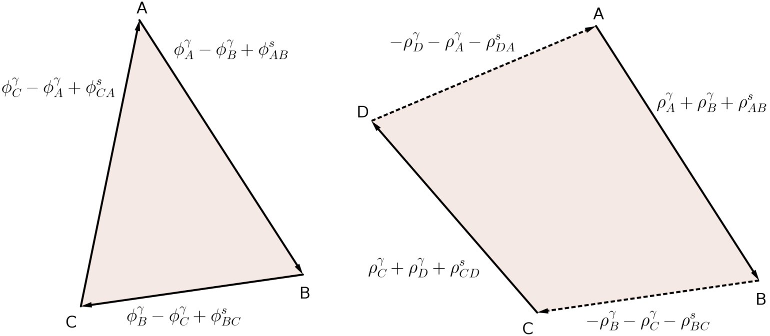
A
$\phi^\gamma_A - \phi^\gamma_B + \phi^s_{AB}$
$\phi^\gamma_C - \phi^\gamma_A + \phi^s_{CA}$
B
C
$\phi^\gamma_B - \phi^\gamma_C + \phi^s_{BC}$

Likelihood



A
B
C
$\phi_A^\gamma - \phi_B^\gamma + \phi_{AB}^s$
$\phi_C^\gamma - \phi_A^\gamma + \phi_{CA}^s$
$\phi_B^\gamma - \phi_C^\gamma + \phi_{BC}^s$
A
B
C
D
$-\rho_D^\gamma - \rho_A^\gamma - \rho_{DA}^s$
$\rho_A^\gamma + \rho_B^\gamma + \rho_{AB}^s$
$\rho_C^\gamma + \rho_D^\gamma + \rho_{CD}^s$
$-\rho_B^\gamma - \rho_C^\gamma - \rho_{BC}^s$

Likelihood



$$\phi^{d}_{clos} = P\phi^{d} \qquad \rho^{d}_{clos} = L\rho^{d}$$

Likelihood

$$P\left(\phi^d_{clos}|s\right) \approx \mathcal{G}\left(e^{i\phi^d_{clos}} - e^{i\phi^s_{clos}}, PNP^\dagger\right)$$

$$P\left(\rho^d_{clos}|s\right) \approx \mathcal{G}\left(\rho^d_{clos} - \rho^s_{clos}, LNL^\dagger\right) \quad \text{with} \quad N = \text{diag}\left(\frac{\sigma^2}{|d|^2}\right)$$

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$$s\left(\xi\right) = F_M\left(\xi\right)$$

$$P\left(\xi|\phi^d_{clos}, \rho^d_{clos}\right) \propto P\left(\phi^d_{clos}|s\left(\xi\right)\right) P\left(\rho^d_{clos}|s\left(\xi\right)\right) \ \mathcal{G}\left(\xi, \mathbb{1}\right)$$

Approximate Inference

$$D = (\phi^d_{clos}, \rho^d_{clos})$$

$$KL\left(Q(\xi), P\left(\xi|D\right)\right) = -\int Q(\xi)\log\left(\frac{P\left(\xi|D\right)}{Q(\xi)}\right)\,\mathrm{d}\xi$$

$$Q(\xi) = \mathcal{G}(\xi - m, \mathcal{M}_m^{-1})$$

$$\mathcal{M}_m = -\left\langle \frac{\partial^2 \log\left(P\left(D|\xi\right)\right)}{\partial\xi\;\partial\xi}\right\rangle_{P(D|\xi)}\Bigg|_{\xi=m} + \mathbb{1}$$

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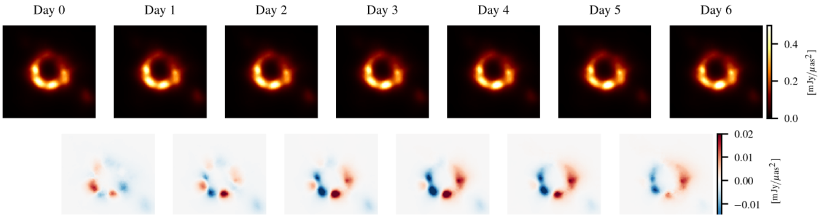
Set of N approximate posterior samples $\{\xi_i\}_i$

$$\langle s\rangle_{P(s|D)} = \langle s(\xi)\rangle_{P(\xi|D)} \approx \frac{1}{N}\sum_{i=1}^{N} s(\xi_i)$$

Results

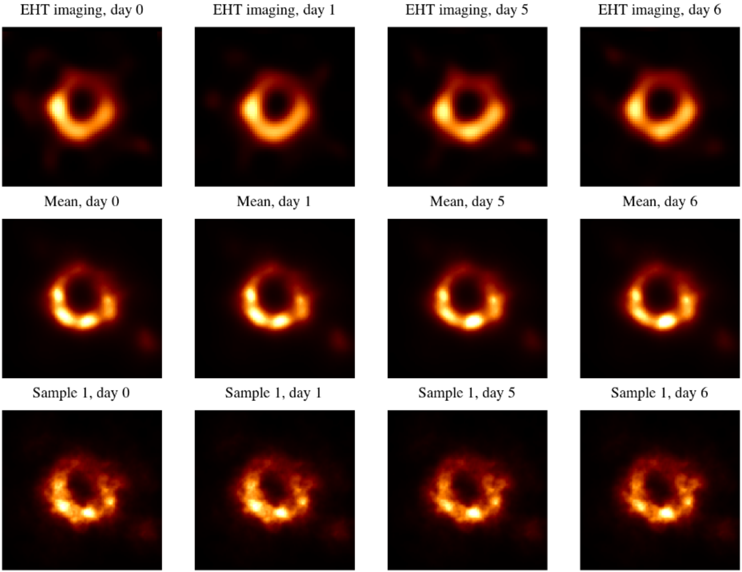
Results



3

[3] Arras et al., "The variable shadow of M87*".

Results



[4]

[4] Arras et al., "The variable shadow of M87*".

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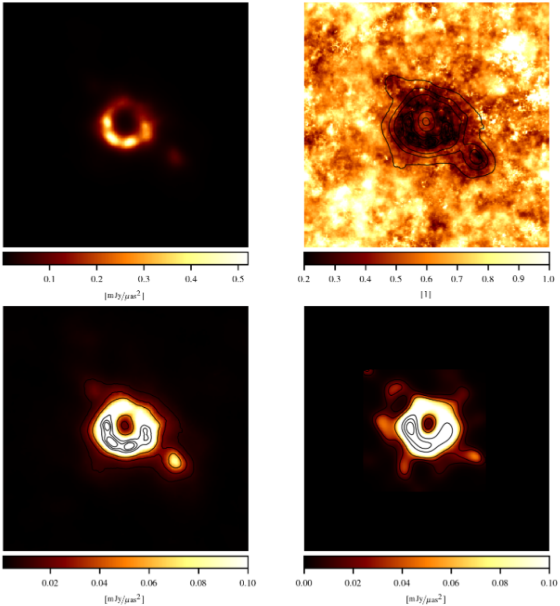
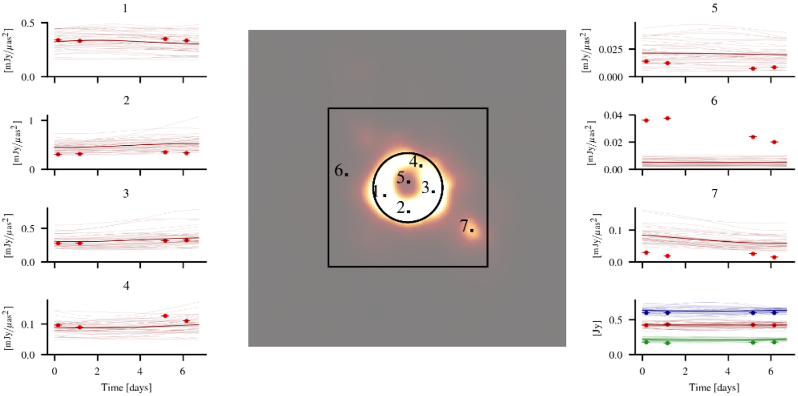


0.1 0.2 0.3 0.4 0.5
[mJy/μas²]
0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
[1]
0.02 0.04 0.06 0.08 0.10
[mJy/μas²]
0.00 0.02 0.04 0.06 0.08 0.10
[mJy/μas²]

Results



[5]

[5] Arras et al., "The variable shadow of M87*".

Results

Day	d (μas)	w (μas)	η ($^\circ$)	A	f_C
EHT-IMAGING					
0	39.3 ± 1.6	16.2 ± 2.0	148.3 ± 4.8	0.25 ± 0.02	8×10^{-1}
1	39.6 ± 1.8	16.2 ± 1.7	151.1 ± 8.6	0.25 ± 0.02	6×10^{-2}
5	40.7 ± 1.6	15.7 ± 2.0	171.2 ± 6.9	0.23 ± 0.03	4×10^{-2}
6	41.0 ± 1.4	15.5 ± 1.8	168.0 ± 6.9	0.20 ± 0.02	4×10^{-2}
OUR METHOD (SAMPLE UNCERTAINTY)					
0	40.2 ± 1.1	13.4 ± 2.4	158.3 ± 4.5	0.28 ± 0.02	1×10^{-4}
1	40.3 ± 1.1	13.3 ± 2.5	157.1 ± 4.1	0.29 ± 0.02	1×10^{-4}
5	40.8 ± 1.2	13.3 ± 2.4	172.8 ± 4.4	0.27 ± 0.03	1×10^{-4}
6	41.0 ± 1.1	13.3 ± 2.5	176.6 ± 4.7	0.27 ± 0.03	1×10^{-4}

Data products

EHT observation data: https://doi.org/10.25739/g85n-f134

Posterior samples[6]: https://doi.org/10.5281/zenodo.3664583

IFT Imaging Pipeline: https://gitlab.mpcdf.mpg.de/ift/vlbi_resolve

[6] Arras et al., "The variable shadow of M87*".